Institute of
Space Sciences

EXCELENCIA
MARÍA
DE MAEZTU

ICREA

Quantitative exploration of pulsar light curve similarity

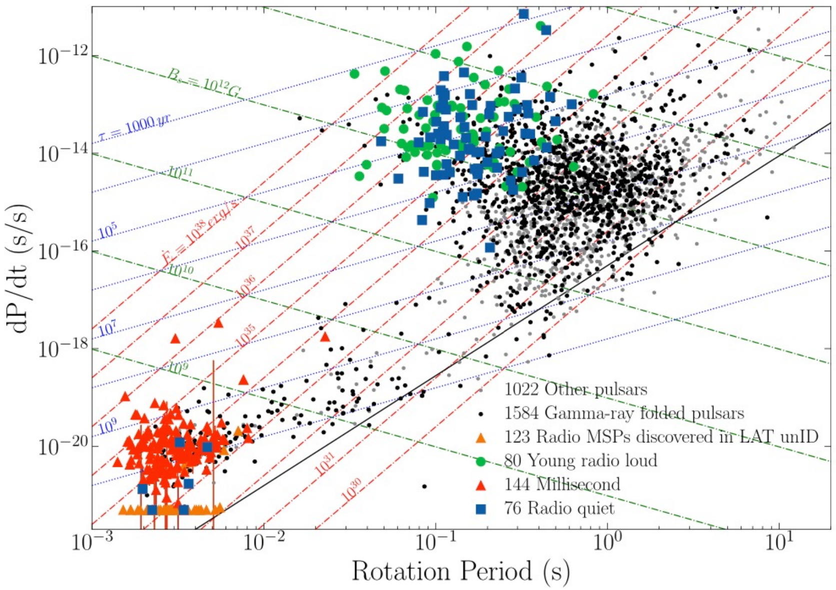
Diego F. Torres

Research done in collaboration with C. R. García

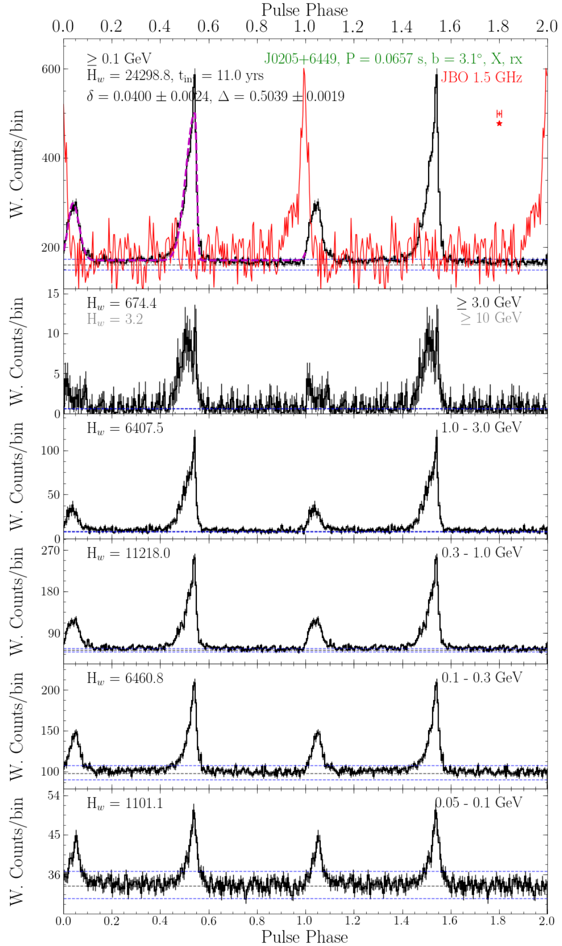
Variety of Pulsar Light curves in the 3PC

Institute of Space Sciences
EXCELENCIA MARÍA DE MAEZTU

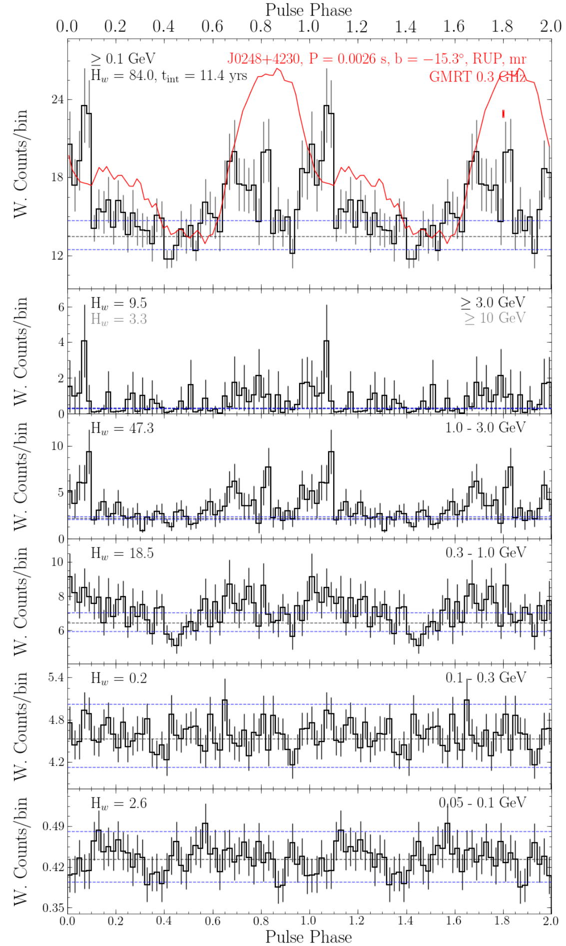
- About 300 pulsars reported
- 3PC light curves sampled with different levels of precision (from 25 to 800)



J0205+6449



J0248+4230



Smith et al. 2024 (3PC)

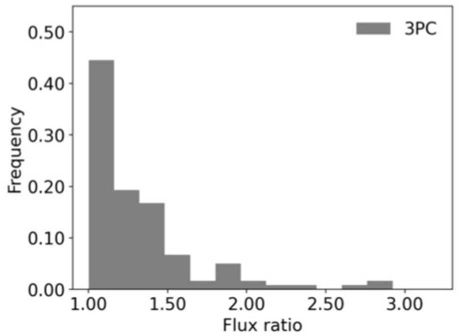
Variety of Pulsar Light curves in the 3PC

Institute of
Space Sciences

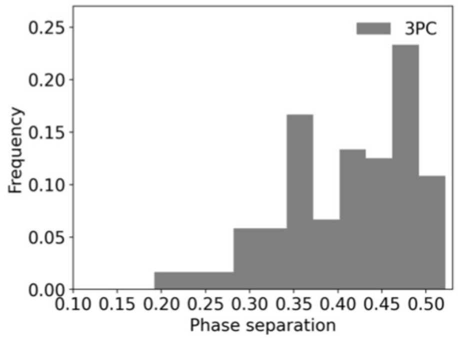
EXCELENCIA
MARÍA
DE MAEZTU

- Light curves described via global features

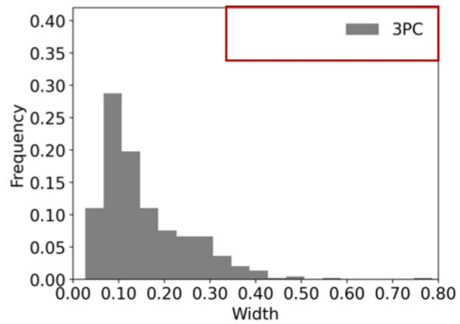
Flux ratio between
two peaks



Phase
separation



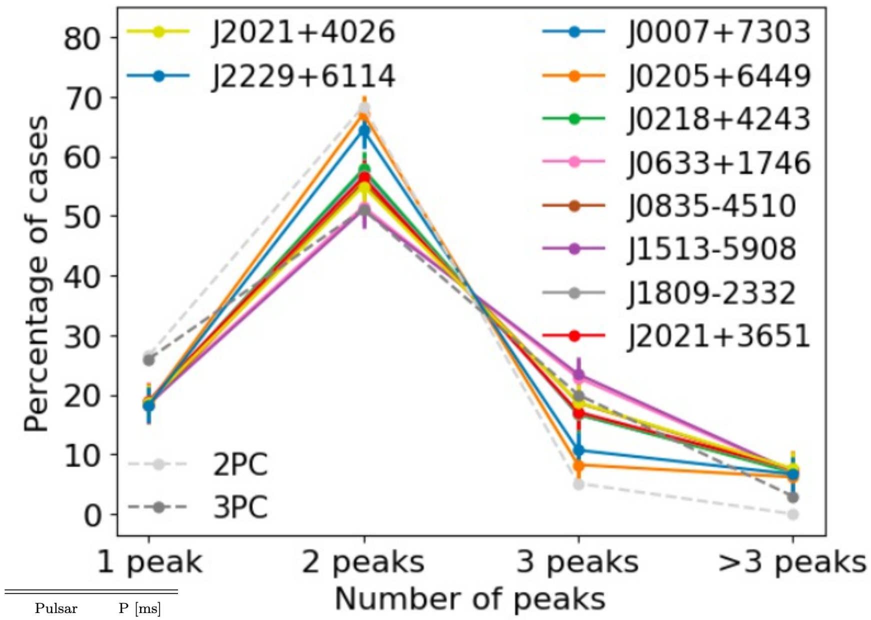
Width
of the peaks



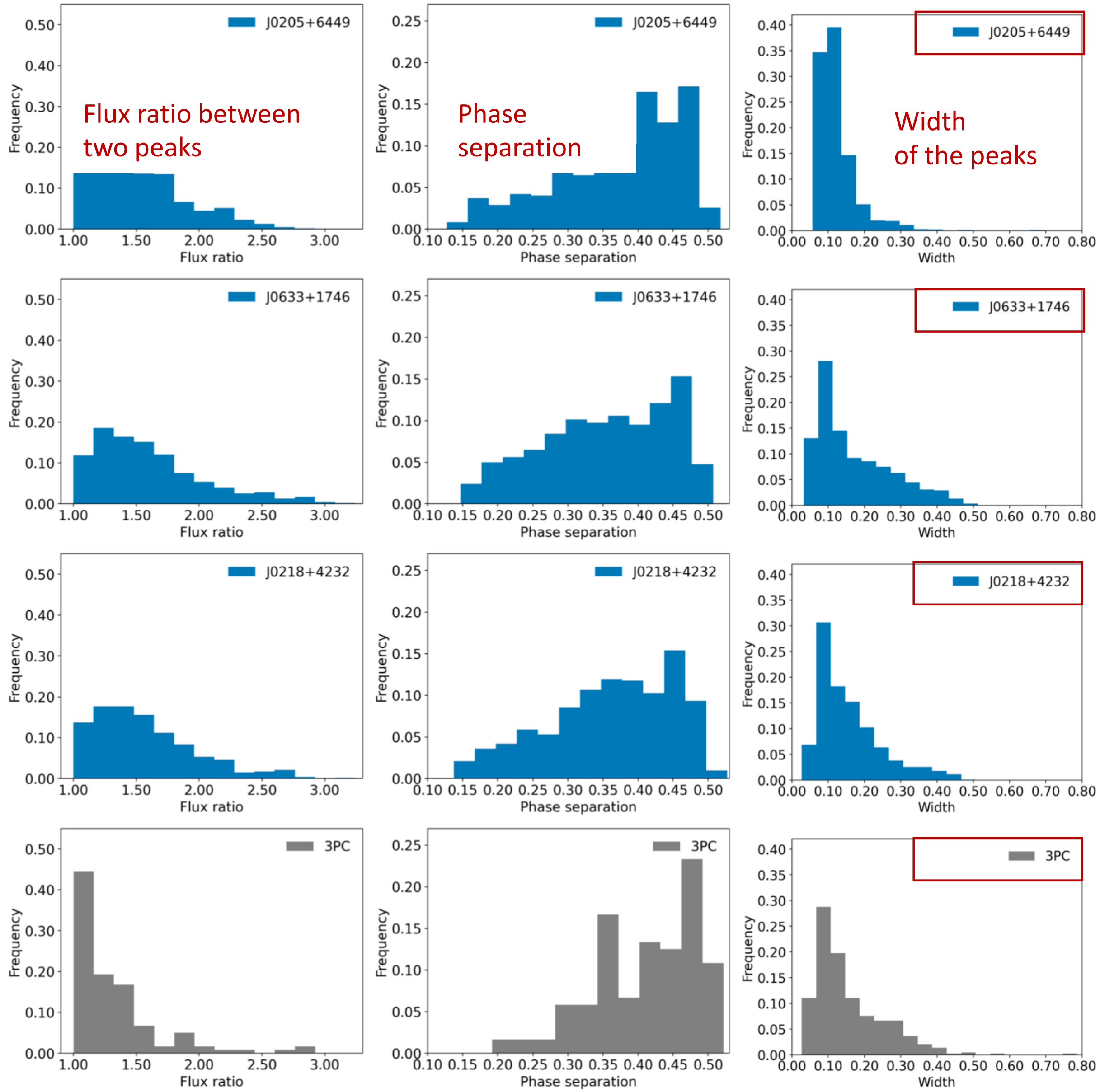
Smith et al. 2024 (3PC)

Light curves global properties

Number (or percentage) of light curves with n peaks



Pulsar	P [ms]
J0007+7303	315.9
J0205+6449	65.7
J0218+4232	2.3
J0633+1746	237.1
J0835−4510	89.4
J1513−5908	151.6
J1809−2332	146.8
J2021+3651	103.7
J2021+4026	265.3
J2229+6114	51.7

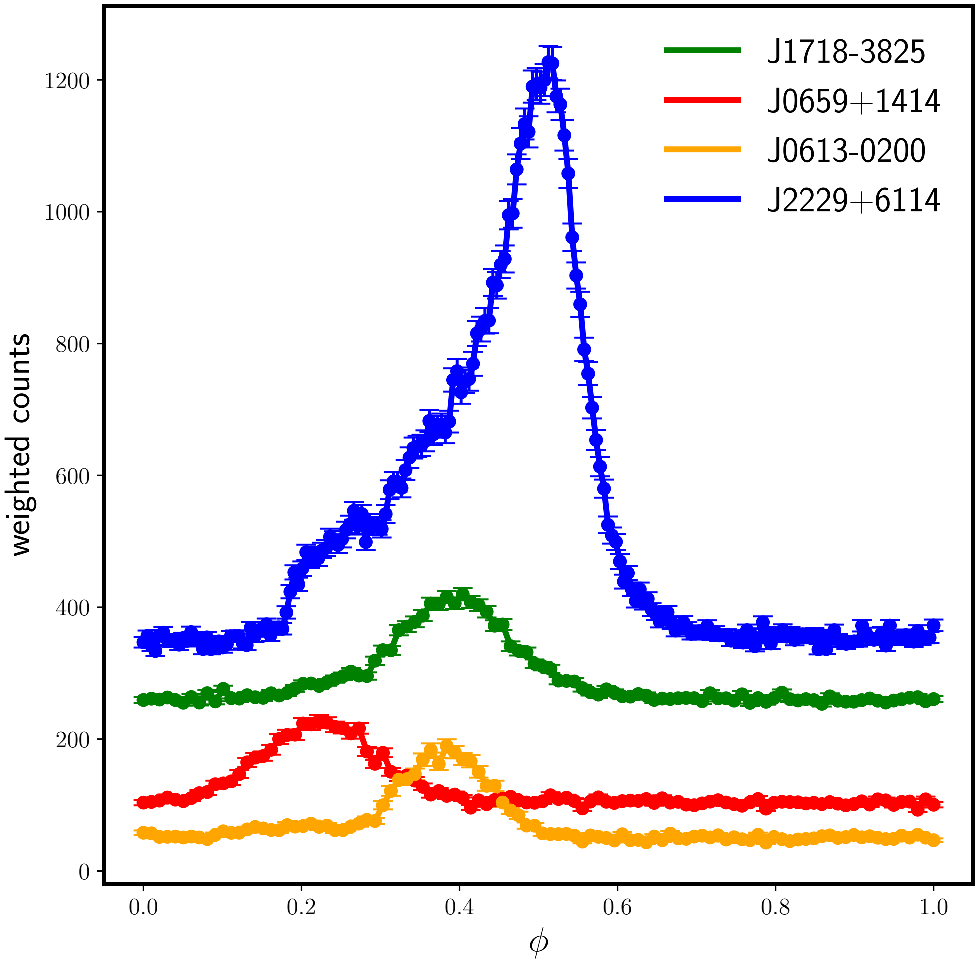


Iñiguez-Pascual, Torres, Viganò, MNRAS 2024 (see talk by D. Viganò)

Variety of Pulsar Light curves in the 3PC

Institute of
Space Sciences

EXCELENCIA
MARÍA
DE MAEZTU



3PC light curves examples,
obtained directly from the catalog

- Very different in flux level and shape, are they?
- Consider the transformation

  Counts → (Counts – Background) / (Max-Background)

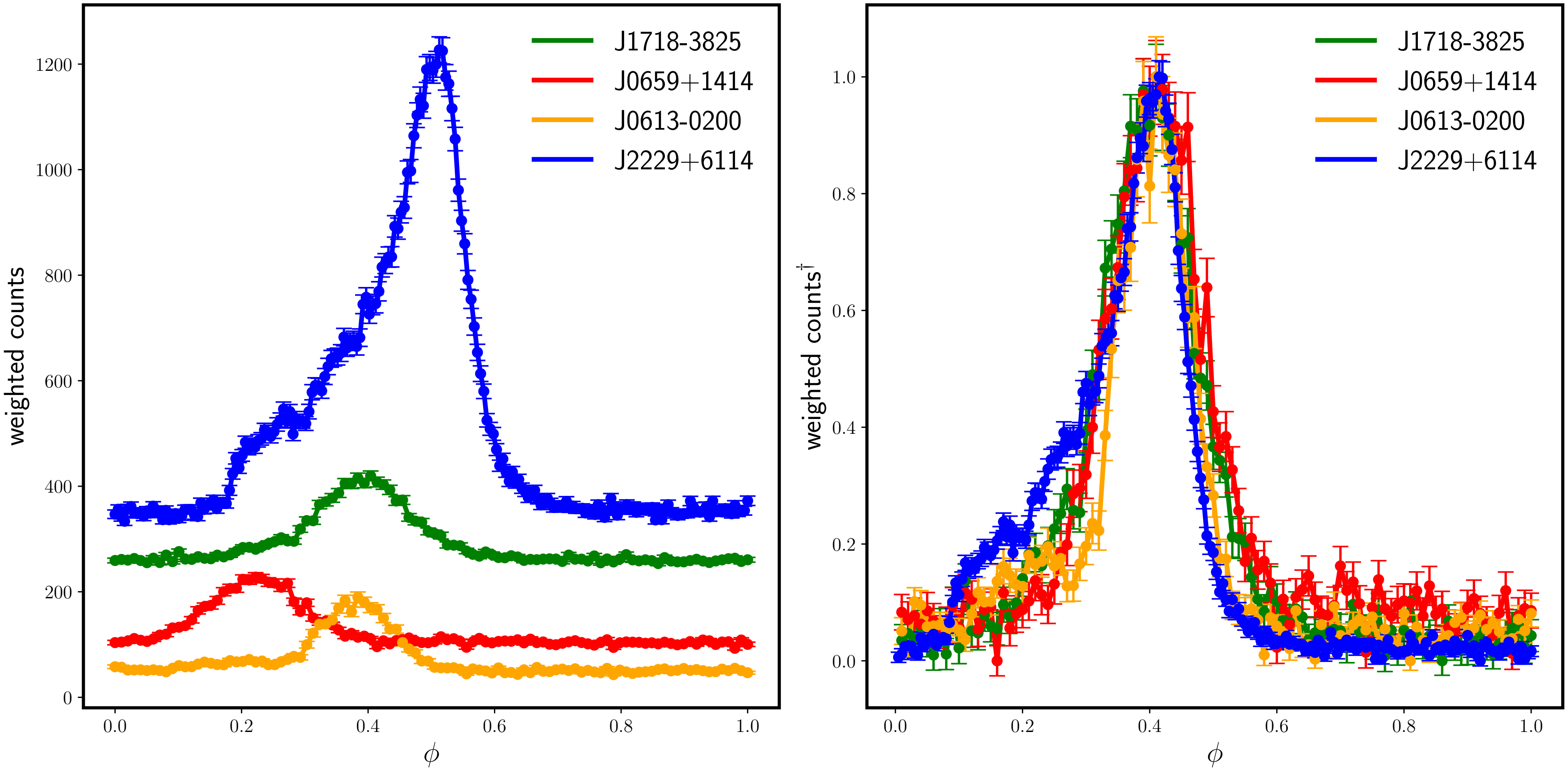
- And all light curves rotations in phase
- Based on:
  - Phase is arbirtray so rotate them until 'alignment'
  - maps all light curves in a range from 0 to 1 in 'counts'
  - emphasizes morphological differences

García & Torres, in preparation

A realization: morphological similarity underlies

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

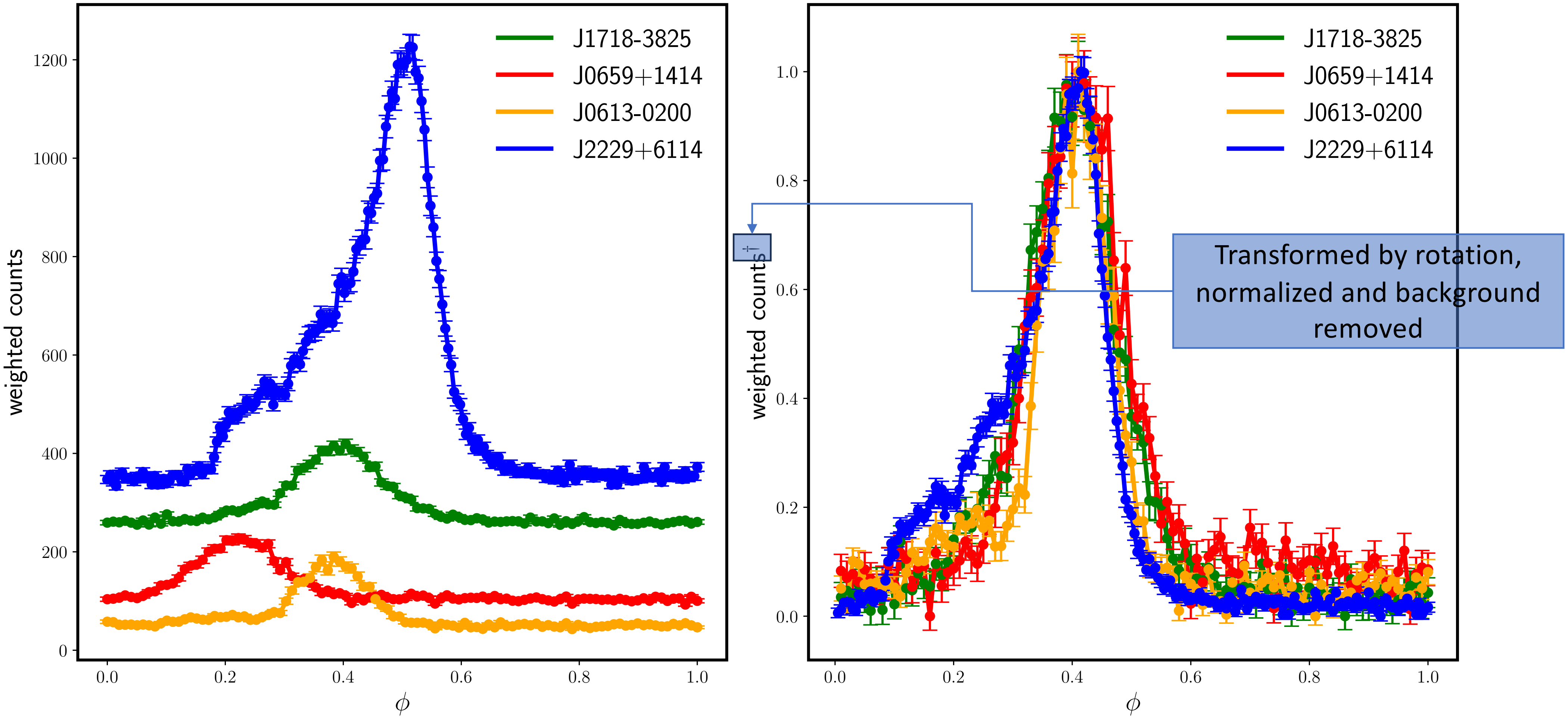


García & Torres, in preparation

A realization: morphological similarity underlies

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

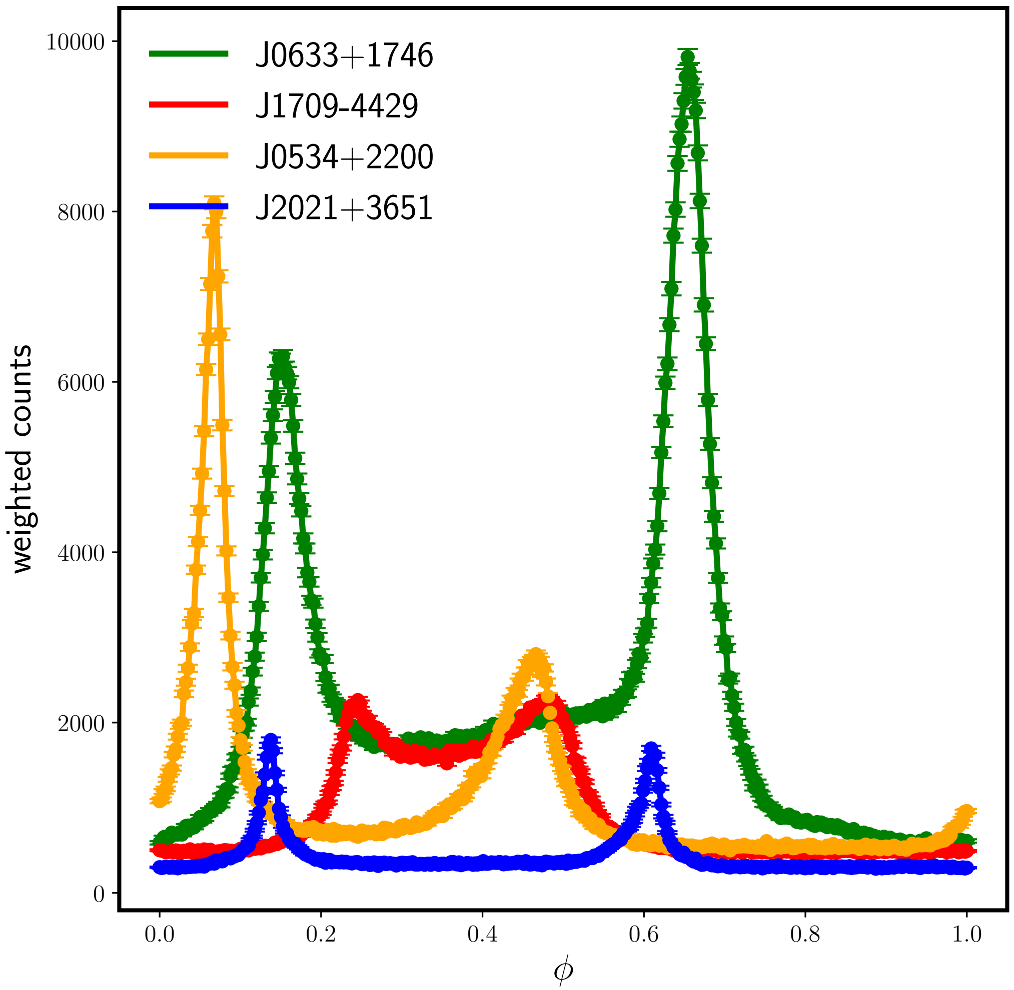


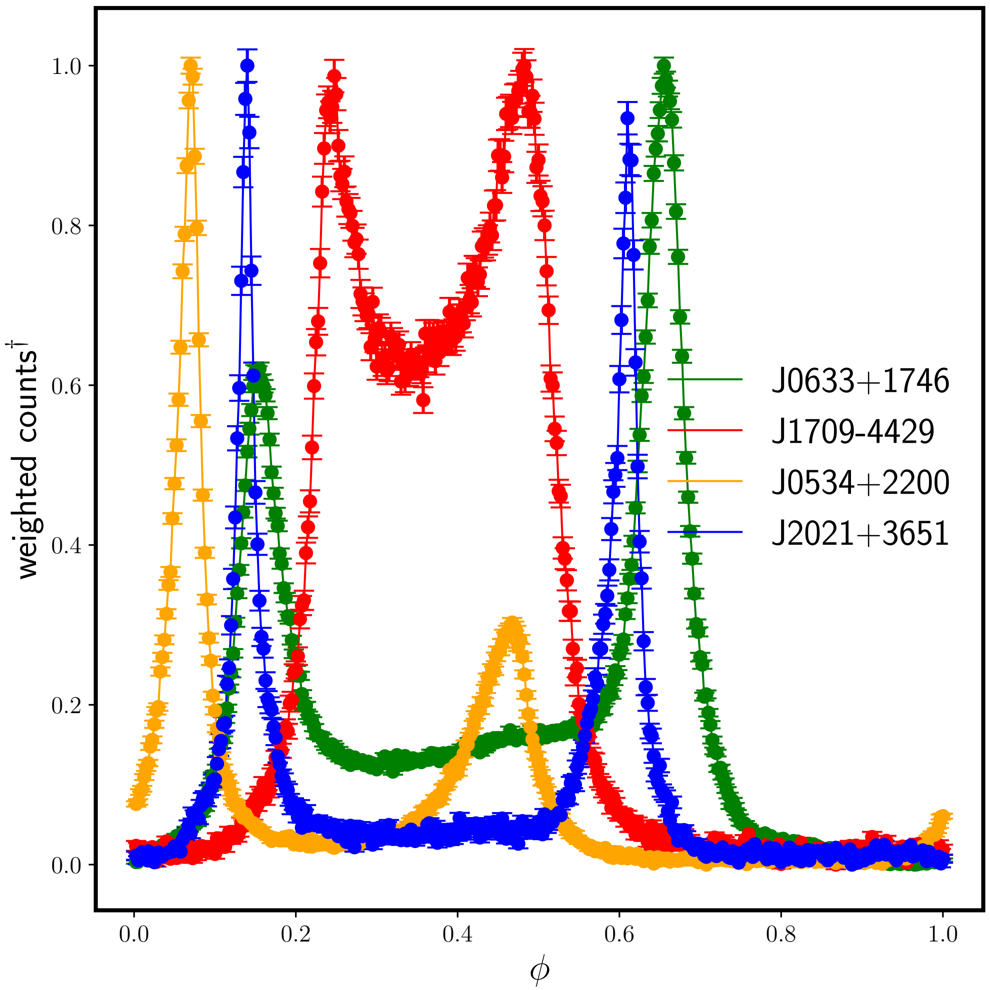
García & Torres, in preparation

Not always of course, there is also dissimilarity

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

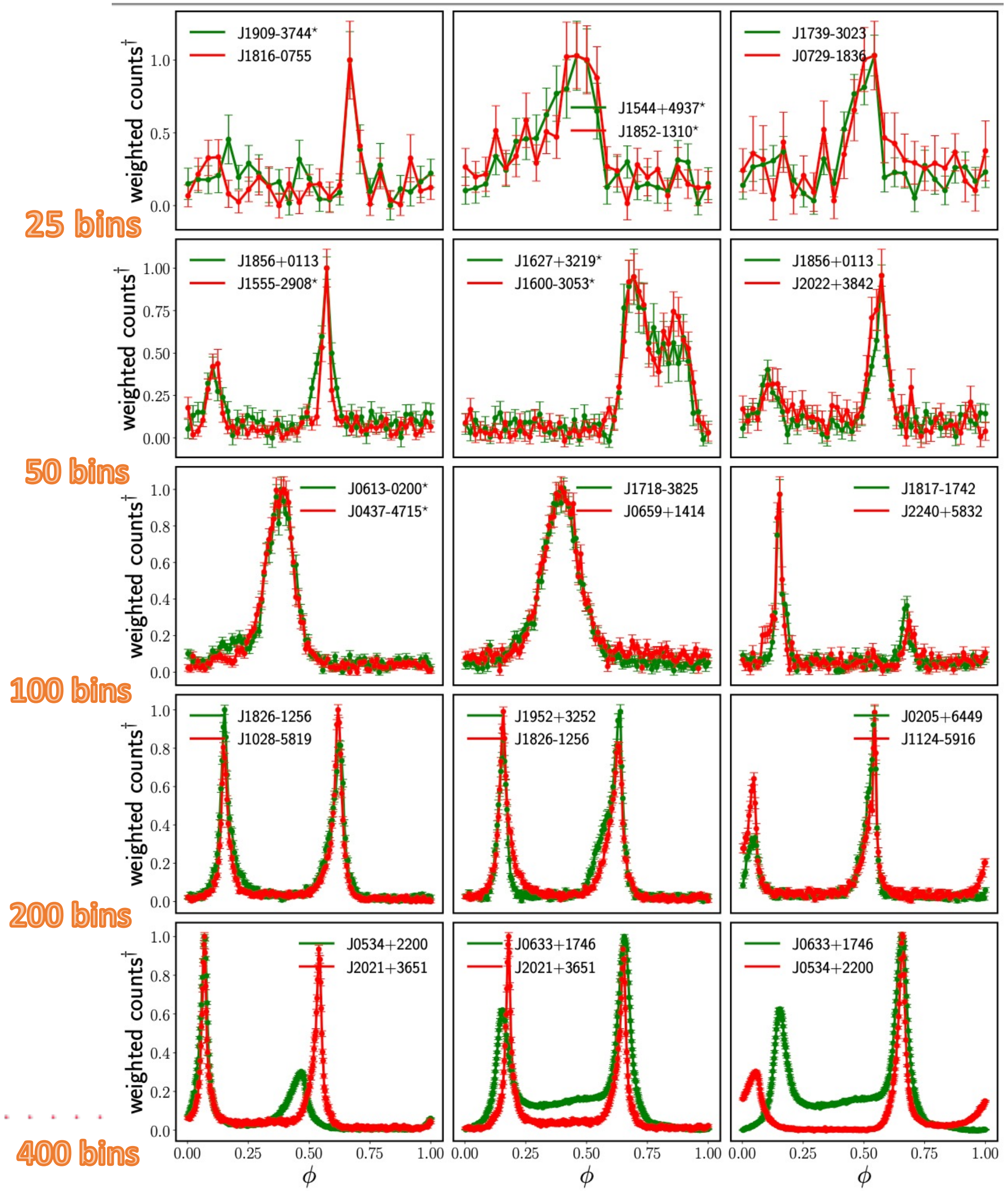




García & Torres, in preparation

Consider Euclidean distance from each pulsar to all others, including all rotations, and rank them

- Arising similarities all over the sample
- Significant matching in many cases
- **But limited**
  - to comparing pulsars having the same number of bins in the light curve, or subjet to a rebinning process to make them so, with the subsequent loss of information
  - Very similar pulsars, not precisely aligned are not singled out as being similar



García & Torres, in preparation

Dynamic time warping: concept

- DTW is an optimization method used to compare time series.
- It works by **dynamically aligning** these time series even with **different sizes**.
- The Euclidean distance (ED) is used once such dynamic alignments (or paths) are established.
- The goal is the optimization of the, so the path with the minimum cost is labelled as *the optimal warping path* and its cost will be *DTW value*.

Dynamic time warping: math

- For this process to be effective we set the next conditions to consider a path:
  - **Boundary condition**.
    - The initial and final elements of the time series must face each other respectively.
  - **Monotonicity condition**.
    - No elements of the series that break the temporal order of the series can be matched against each other.
  - **Continuity condition**.
    - The elements of the series being aligned must be adjacent points, not allowing temporal jumps.
- The resolution for this optimal problem can be seen via:

$$DTW(X,Y) = min_{\pi \in A(X,Y)} \Big(\sum_{(i,j)\in\pi} d(x_i, y_j)^2 \Big)^{\frac{1}{2}}$$

$$\begin{cases} X, Y\text{: } time\ series \\ A\text{: } set\ of\ paths \\ \pi\text{: } path \\ d\text{: } ED \end{cases}$$

Dynamic time warping: practice & simple example

- We define the time series as:

$$ts_1 = 3, 1, 2, 2, 1$$
$$ts_2 = 2, 0, 0, 3, 3, 1, 0.$$

- The ED matrix (E) between the ts_1 and ts_2:

$$E(d(i,j)^2) = \begin{bmatrix} 1 & 9 & 9 & 0 & 0 & 4 & 9 \\ 1 & 1 & 1 & 4 & 4 & 0 & 1 \\ 0 & 4 & 4 & 1 & 1 & 1 & 4 \\ 0 & 4 & 4 & 1 & 1 & 1 & 4 \\ 1 & 1 & 1 & 4 & 4 & 0 & 1 \end{bmatrix}$$

- The cumulative cost matrix through:

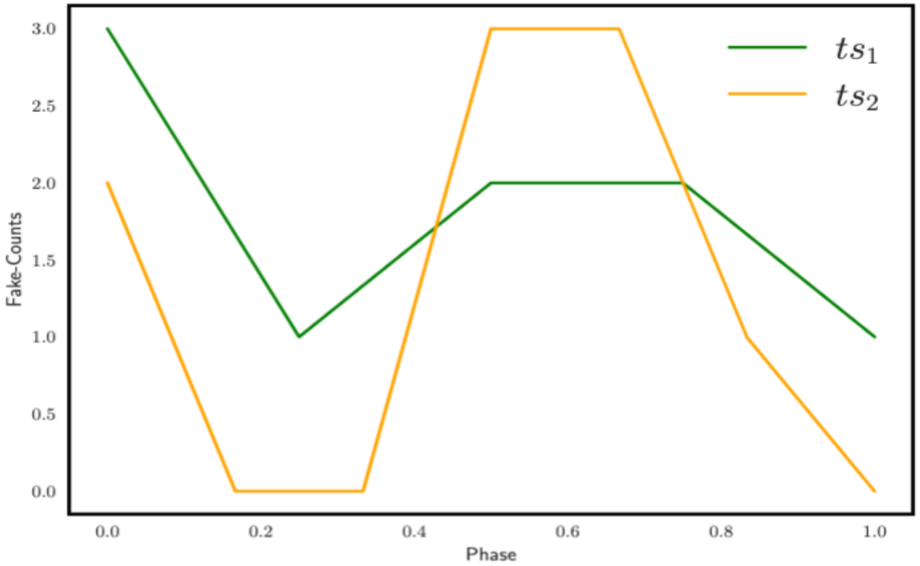
$$D(i,j) = E(i,j) + min\big(D(i-1,j), D(i,j-1), D(i-1,j-1)\big).$$

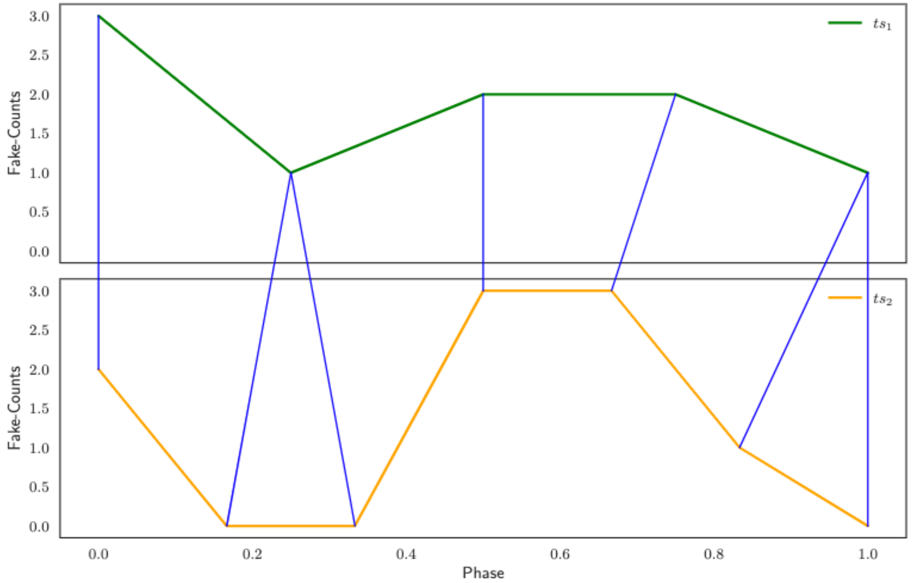
$D =$

- For instance, the value seen in row 2 and column 2, *D(2,2)= E(2,2) + min(D(1,2), D(2,1), D(1,1)) = 1 + min(10,2,1) = 2.*

- The DTW value is the total cost of the blue line, according to the *E*-matrix: $\sqrt{1+1+1+1+1+0+1} = 2.45$

Dynamic time warping: example in a figure





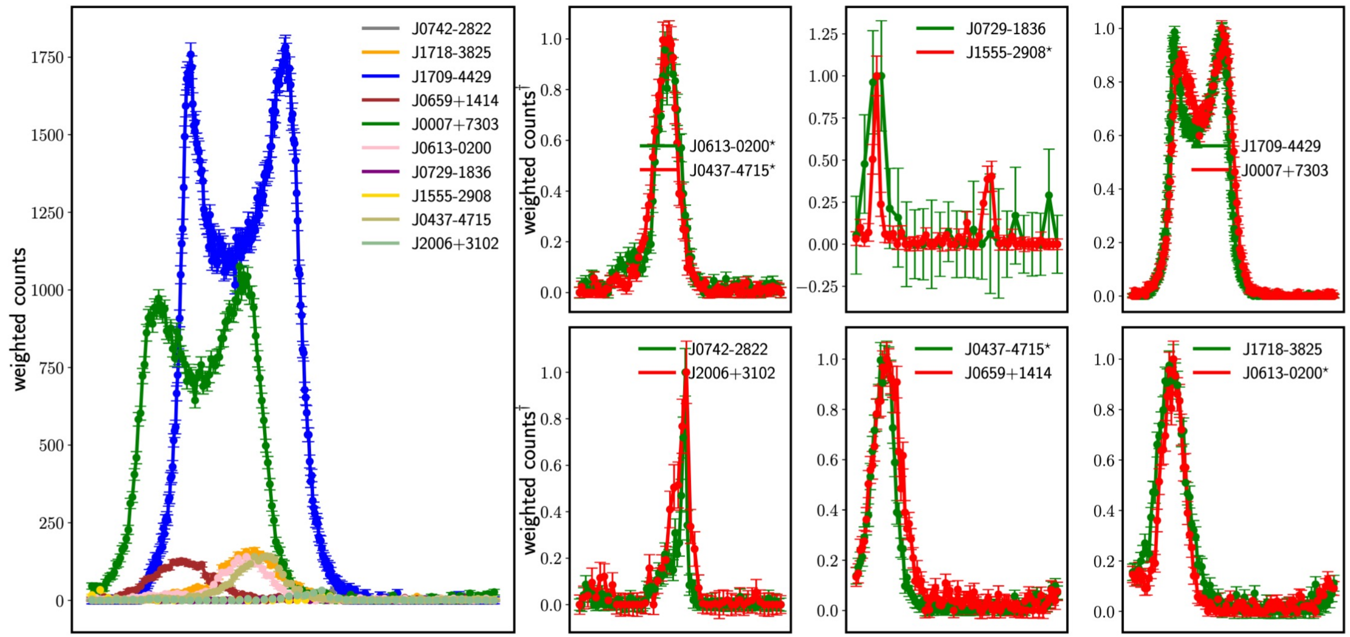
The *optimal warping path* is represented by the blue lines where each one defines a distance.

A comparison devoid of limitations

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

- A technique useful to find similarities beyond the simple alignment (which is what the Euclidean distance quantifies)
- It associates morphological structures, despite they do not happen at exactly the same phase
- Can be applied to light curves of any number of bins, without rebinning
- Provides a number, that can be used to quantify similarity



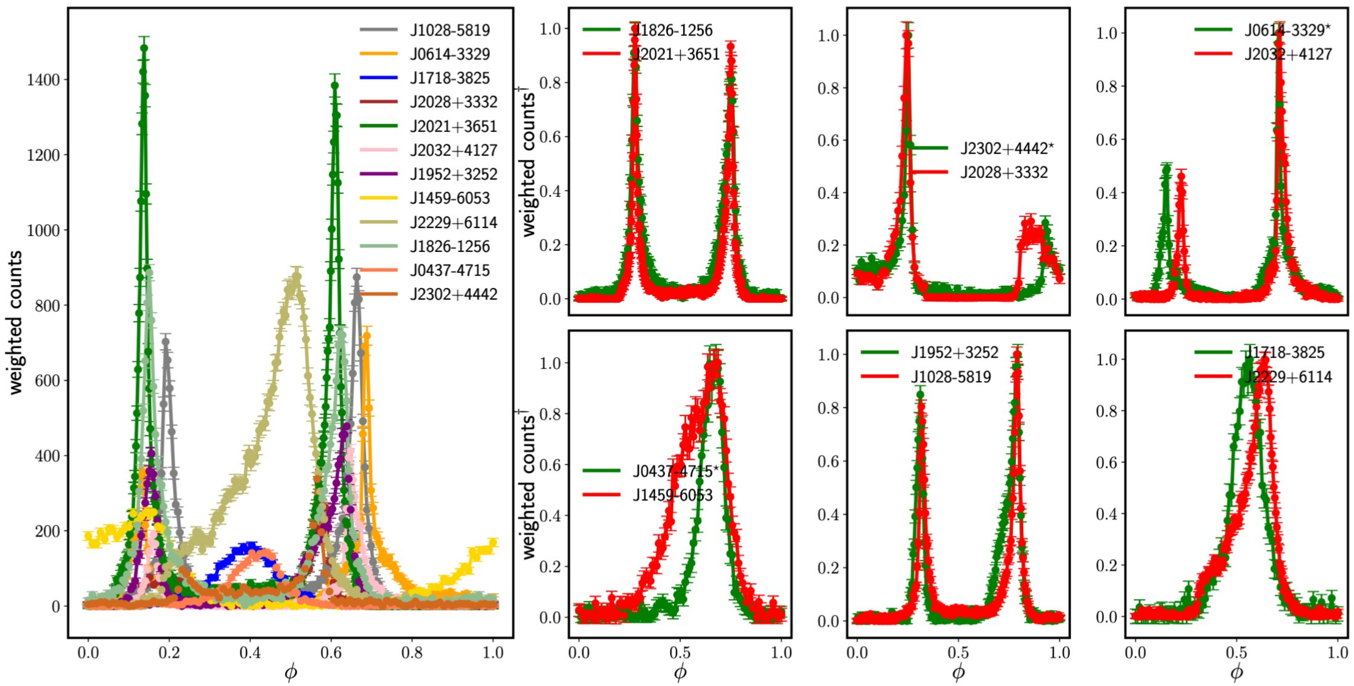
García & Torres, in preparation

A comparison devoid of limitations

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

- A technique useful to find similarities beyond the simple alignment (which is what the Euclidean distance quantifies)
- It associates morphological structures, despite they do not happen at exactly the same phase
- Can be applied to light curves of any number of bins, without rebinning
- Provides a number, that can be used to quantify similarity



A comparison devoid of limitations

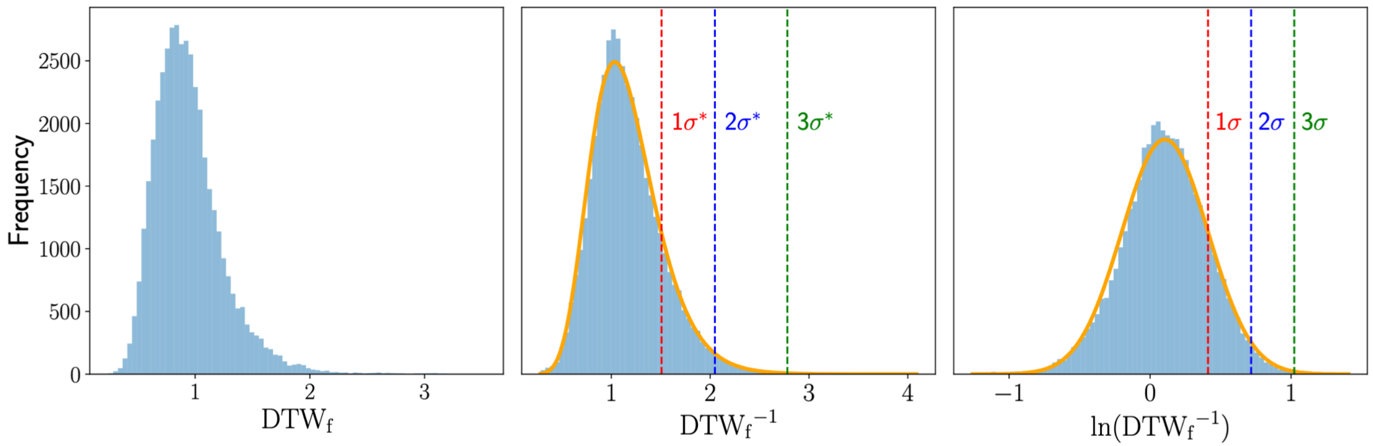
Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

- Computing the dynamic distance between all pulsars in the sample, and all its rotations, one obtain its distribution: a global view of how similar/dissimilar pulsar light curves are

- A well behaving quantitative similarity estimator

- Permits to define intervals of confidence for the similarity of light curves

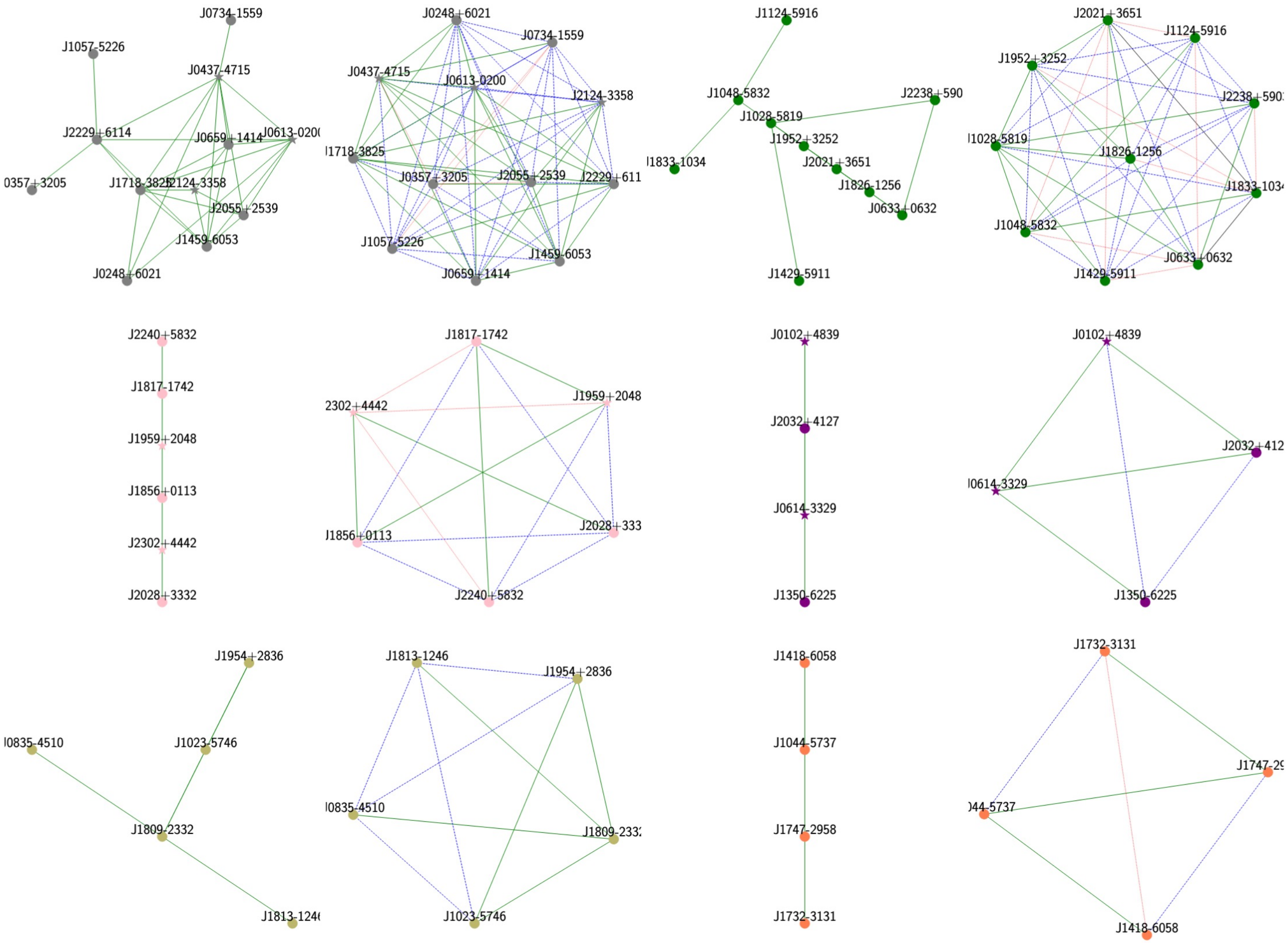


Left panel: Distribution of the DTW_f obtained after comparing the 294 light curves with each other giving rise to 43071 values.
Center panel: Distribution of the inverse of DTW_f (so that the larger DTW are the more similar pairs of light curves). The orange line shows the log-normal distribution, identified as the best fit. The dashed vertical lines denoted with $1\sigma*$ (red), $2\sigma*$ (blue), and $3\sigma*$ (green)
Right panel: Distribution of the natural logarithm of $\mathrm{DTW}_f{}^{-1}$. The orange line shows the normal distribution. The dashed vertical lines denoted with 1σ (red), 2σ (blue), and 3σ (green) represent the intervals of the distribution according to the known empirical rule.

We can now cluster the light curves using similarity

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU



García & Torres, in preparation

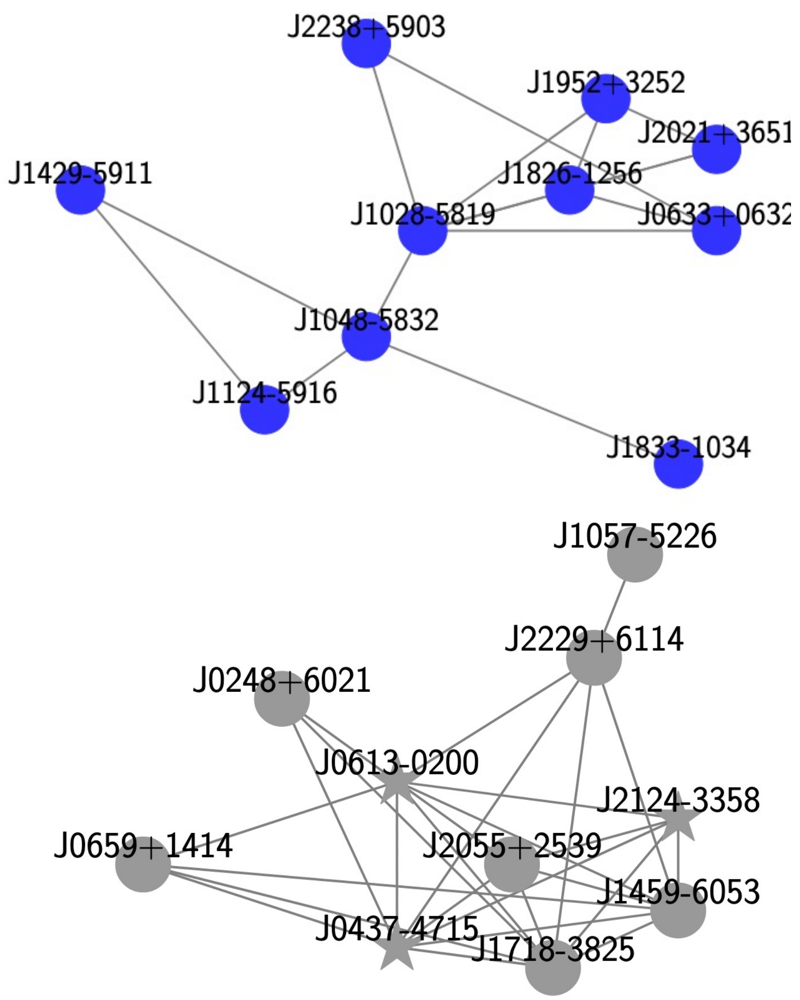
MSPs are denoted with a star, while pulsars with P>10 ms are shown with a circle.

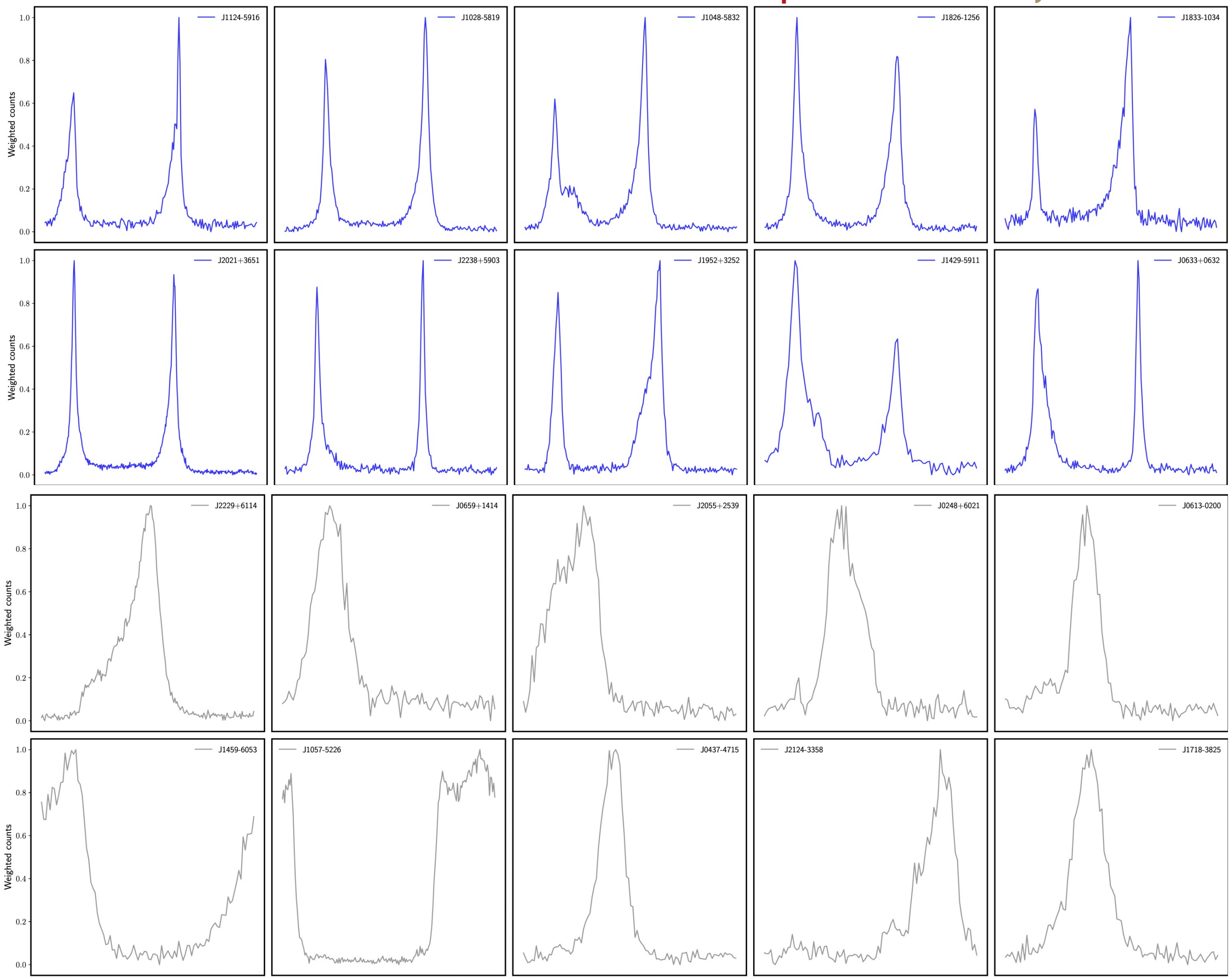
Black edges indicate similarity is in the range 1-2 sigma, blue 2-3 sigma and green >3 sigma.

We can now cluster the light curves using similarity

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU





García & Torres, in preparation

Conclusions

Pulsar similarity can now be phenomenologically quantified

- Light curves look more different than they really are, in many cases.
- Euclidean and DTW provide quantitative estimators of their morphological similarity
- Euclidean distance is sub-efficient:
  - Cannot deal with light curves of different sizes
  - Cannot assign similarity to light curves that really are if they are slightly displaced (e.g., two peaks with slightly different separation)
- Clustering via light curve similarity can be used to compare with physical and spectral properties of pulsars
- Applications of this methodology go much beyond pulsar light curves
  - TPs, magnetar and GRB light curves, etc.

García & Torres, in preparation

Thank you for your attention

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

CSIC

Institute of Space Sciences

EXCELENCIA MARÍA DE MAEZTU

ICREA