CHEMICAL EVOLUTION FORMULATION

ANALYTICAL MODELS

The equations of chemical evolution

$$\frac{dM}{dt} = f$$

$$\frac{dM_s}{dt} = Y - E$$

$$\frac{dM_g}{dt} = -Y + E + f$$

$$M = M_g + M_s$$

M = total mass of the system

M_s = total stellar mass

M_g = gas mass

Ψ = Star Formation Rate, SFR

E = mass ejection rate from stars

f = $f_{in} - f_{out}$ = net flux of gas

The evolution of the gas metallicity

$$\frac{d(ZM_g)}{dt} = -Z\Psi + Z_{in} f_i + -Z_{out} w_{out} + E_z$$

$Z\ \Psi$ $\equiv$ mass of metals incorporated to newly forming stars

$Z_{in} f_{in}$ $\equiv$ mass of metals in the inflowing gas

$Z_{out} F_{out}$ $\equiv$ mass of metals in the outflowing gas

E_z $\equiv$ amount of metals ejected by evolving stars

The rate of metal ejection

- The ejection of matter by stars takes place during the post main sequence evolutionary phase.
- The ejection rate is related to the number of stars which leave the MS at a given time.

$$E(t) = \int_{m_t}^{m_u} (m - w_m)\,\Psi(t - \tau_m)\,\phi(m, t - \tau_m)\,dm$$

τ_m ≡ mean MS life time of stars of initial mass m
$\phi\,(m, t-\tau_m)$ ≡ IMF at the time of formation of stars with initial mass m
$\Psi(t-\tau_m)$ ≡ mass of stars formed in the $(t-\tau_m)$
w_m ≡ remnant of a star of mass m
$(m-w_m)$ ≡ mass ejected by a star of mass m

The mass of ejected gas

$$E_Z(t) = \int_{m_t}^{m_u} m p_{z,m} \Psi(t-\tau_m) \phi(m, t-\tau_m)\, dm +$$

$$\int_{m_t}^{\infty} (m - w_m - m p_{z,m}) Z(t-\tau_m) \Psi(t-\tau_m) \phi(m, t-\tau_m)\, dm$$

- The first term of this equation represent the amount of new elements synthesised inside stars.
- The second term represents the amount of metals that were already in the gas from which a new stars formed.
- $p_{z,m}$ is the amount of element z created inside stars of mass m and $Z(t-\tau_m)$ is the metallicity if the gas at the time of formation of new stars.

Primary and secondary elements

- In the previous equations p_z represents the stellar yield of a primary element.
- A primary element is entirely formed inside stars from the H and He initially present in them.
- On the contrary, a secondary element is synthesized from a seed element previously existing in the stellar interior.
- Secondary element can be synthesised only in second (and higher) generation stars.
- The mass ejection of a secondary element, X, is:

$$E(t) = \int_{m_t}^{m_u} m p_X Z(t-\tau_m) Y(t-\tau_m) f(m, t-\tau_m)\, dm +$$

$$+ \int_{m_t}^{m_u} (m - w_m) X(t-\tau_m) Y(t-\tau_m) f(m, t-\tau_m)\, dm$$

- This equation is similar to the one written for E_Z, but contains two terms: the first one represents the amount of element synthesised and the second one the amount of recycled element.
- The first term depends on the pre-existing metals in the star.

The equation of conservation of metals

$$<Z_s> M_s + Z M_g = \int_0^t \int_{m_t}^{m_u} m\, p_{z,m}\, \Psi(t' - \tau_m)\, \phi(m, t' - \tau_m)\, dt'\, dm$$

where $<Z_s>$ is the average content of metals in stars.

Analytical approximations

- Therefore, to know the chemical evolution of a system requires the resolution of a coupled differential equation system with four time-dependent variables:
  - Ψ , ϕ, f , Z_s
- The problem can be somewhat simplified under the following assumptions:
  - The IMF does not depend on time $\Rightarrow \phi(m, t\text{-}\tau_m) \equiv \phi(m)$
  - Instantaneous recycling approximation (IRA): The stars with masses $m \leq m_1$ are on the MS and live forever and the stars with masses $m > m_1$ dye a soon as they are formed $\Rightarrow$
    - $(t\text{-}\tau_m) \equiv t$ in the equations for calculating the ejected gas rate and the conservation of metals, and
    - $m_t \equiv m_1$ in the mass limits for integration.

The Instantaneous Recycling Approximation (IRA)

- In this framework, two definitions are made:
  - The **Returned Mass Fraction**: $R = \int_{m_1}^{\infty} (m - w_m)\phi(m)\,dm$

    that is, the amount of mass that a star generation returns back to the ISM.

  - The net **Yield** of element i $\quad y_i = \dfrac{1}{1-R}\int_{m_1}^{\infty} m p_{z,m}\phi(m)\,dm$

    that is the mass fraction of the element i newly produced by a generation of stars relative to the fraction of mass locked inside remnants and never dying low mass stars ($m \leq m_1$).

- If the IFM is assumed universal, R and y_i are constants that depend only on stellar evolution.

Time independent IMF and IRA

$$\frac{dM}{dt} = f_{in} - f_{out}$$

$$\frac{dM_g}{dt} = -(1-R)Y + \left(f_{in} - f_{out}\right)$$

$$\frac{dM_s}{dt} = Y - E = Y - RY = Y(1-R)$$

$$\frac{d(ZM_g)}{dt} = -ZY(1-R) + yY(1-R) + Z_{in}f_{in} - Z_{out}f_{out}$$

Time independent IMF and IRA

$$E(t) = \Psi(t)\int_{m_1}^{m_u}(m - w_m)\phi(m)\,dm = \Psi(t)R$$

$$\frac{d(ZM_g)}{dt} = Z\frac{dM_g}{dt} + M_g\frac{dZ}{dt} \Rightarrow$$

$$M_g\frac{dZ}{dt} = y\Psi(1 - R) + (Z_{in} - Z)f_{in} - (Z_{out} - Z)f_{out}$$

Time independent IMF and IRA: metal conservation

- For a primary element:

$$E_Z(t) = RZ(t)Y(t) + y_z(1-R)\left[1-Z(t)\right]Y(t)$$

- For a secondary element

$$E_X(t) = YZ(1-R)y_X + RXY - y_X(1-R)XY =$$
$$= Y(1-R)y_X(Z-X) + YXR$$

- In many cases, X<<Z and we can assume Z-X $\approx$ Z.
- The equation of conservation of secondary elements is then:

$$M_g \frac{dX}{dt} = Y(1-R)(Z-X)y_X + (X_{in} - X)f_{in} - (X_{out} - X)f_{out}$$

Analytical estimate of yields

- The metals produced during the total life of the system are

$$< Z_s > M_s + Z M_g = \int_0^t \int_{m_t}^{m_u} m p_{z,m} \Psi(t' - \tau_m) dt' dm =$$

$$= \{IRA\} = \int_0^t \Psi(t) dt \int_{m_1}^{\infty} m p_z \phi(m) dm = (1 - R) y_z \Psi_T$$

where Ψ_T is the total SFR during the history of the galaxy.

- Taking into account that

$$\frac{dM_s}{dt} = (1 - R)\Psi \Rightarrow \int_0^t \frac{dM_s}{dt} dt = \int_0^t (1 - R)\Psi \, dt = (1 - R)\Psi_T$$

Analytical estimate of yields

- Therefore $$< Z_s > = \frac{(1-R)y_z Y_T}{(1-R)Y_T} - \frac{M_g Z}{M_s} = y_z - \frac{M_g Z}{M_s}$$
- Defining the gas fraction -- μ -- by $$\mu = \frac{M_g}{M_s + M_g}$$

  we have: $$< Z_s > = y_z - \left(\frac{\mu}{1-\mu}\right) Z$$

- For a given region, if the average stellar metallicity, the present ISM metallicity and the gas fraction can be known, then the net yield of a given element can be estimated.
- This result shows that **$<Z_s> \rightarrow y_z$ when $\mu \rightarrow 0$.**
- For the SN, μ ~0.05 y el yield $y_z \approx 0.8\ Z_\odot$

Estimating the Star Formation History (SFH) of the SN

- To estimate the SFH of the SN we can write:

$$Y = \frac{1}{1-R}\frac{dM_s}{dt} = \frac{1}{1-R}\left(\frac{d\log M_s}{d\log Z}\right)\left(\frac{d\log Z}{dt}\right)$$

- The measurements of the metallicity of the stars in the SN gives the stellar mass as a function of Z.
- The Luminosity of F stars in the SN compared to F stars in the ZAMS (zero age main sequence) together with their metallicities provides the age-metallicity relation – *dlogZ/dt* – .
- This exercise shows that simple analytical approximations already predict many more stars of low metallicity in the SN than observed $\Rightarrow$ ***"G dwarf problem"***

The G dwarf problem

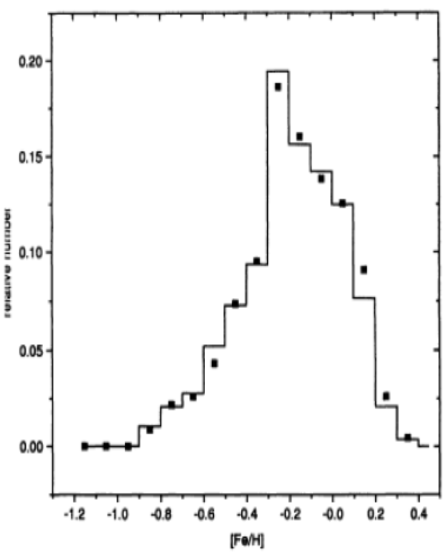


Figure 2. Metallicity distribution of 287 dwarf stars with spectral types in the range G0 – G9 (continuous line), and 231 dwarfs of spectral types G2 – G9 (squares).

[Rocha-Pinto & Maciel 1996, *MNRAS*, **279**, 44

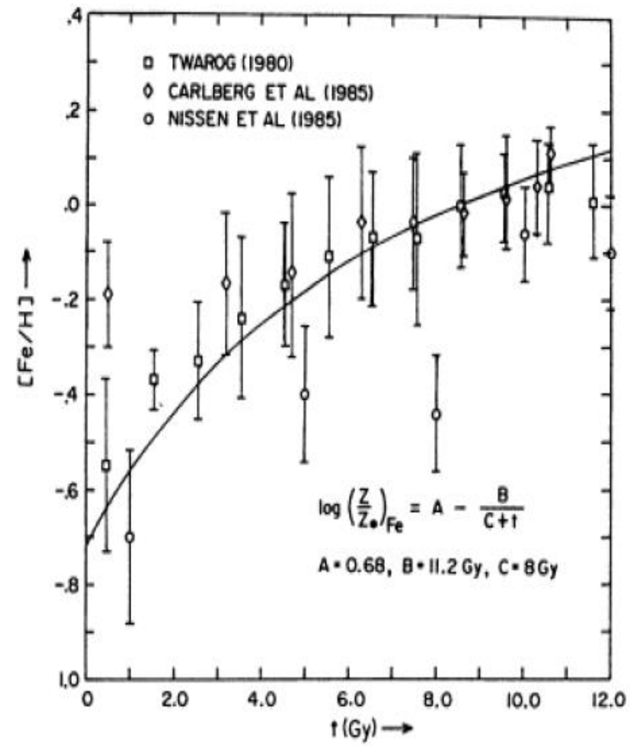


Figure 1 The age-metallicity relation for the local dwarf stars of the thin disk with the fitted curve given by Equation 2.

The simple model

- A closed box system (there is no exchange of matter with the exterior) + IRA + instantaneous mixing.
- f=0 $\Rightarrow$ $\begin{cases} M_g \dfrac{dZ}{dt} = yY(1-R) \\ \dfrac{dM_g}{dt} = -(1-R)Y \end{cases}$
- Dividing one equation by the other: $M_g \dfrac{dZ}{dt} \Big/ \dfrac{dM_g}{dt} = -y_z$
- y_z is given by stellar evolution and is, in principle, constant
- Initially, $M_{g0}=M$ and $Z_0=0$, hence $\int_{Z_0}^{Z_1} dZ = -y_z \int_{M_{g0}}^{M_{g1}} \dfrac{dM_g}{M_g} \Rightarrow Z_1 - Z_0 = -y_z \, ln\left(\dfrac{M_{g1}}{M_{g0}}\right)$

$$Z = -y_z \, ln\left(\frac{M_g}{M}\right) = y_z \, ln\left(\frac{M}{M_g}\right)$$

with solution: $\boxed{Z = y_z \, ln\, m^{-1}}$

The effective yield

- In relation to the simple model of chemical evolution we can define the "***effective yield***" as the yield a system would have if it evolved as a closed box and would be described by the simple model.

$$y_{z,eff} = \frac{Z}{ln(1/m)}$$

- For open systems $y_{z,eff} < y_z$.
- The only way to increase y_z would be to assume an IMF wighted towards massive stars.

Failures of the simple model

- Let us consider the cumulative distribution of the stars form up to the present time. Observationally, these are stars of type G and later.
- The fraction of stars formed when the gas fraction was greater than or equal to μ is:

$$\frac{M_s}{M_{s1}} = \frac{1-\mu}{1-\mu_1}$$

- These stars were formed when the gas metallicity was:

$$Z_g \leq y_z \ln \mu^{-1}$$

- The fraction of stars with metallicities smaller than or equal to Z – $S(Z)$ – is:

$$S(Z) = \frac{M_s}{M_{s1}} = \frac{1-e^{-Z/y}}{1-\mu_1}$$

- We can eliminate the yield using $Z = y_z \ln \mu_1^{-1}$ to obtain:

$$S(Z) = \frac{1-\mu_1^{Z/Z_1}}{1-\mu_1}$$

The G dwarf problem

- In the SN $Z_g \sim 0.7\, Z_\odot$
- $M_g(0) = \underbrace{M_*(\text{present})}_{40 M_\odot/\text{pc}^2} + \underbrace{M_g(\text{present})}_{10 M_\odot/\text{pc}^2}$
- $Z(0)=0 \Rightarrow y_z = 0.43\, Z_\odot$

$$\frac{M_*(<0.25 Z_{sun})}{M_*(<0.7 Z_{sun})} = \frac{1 - e^{-0.25 Z_{sun}/y_z}}{1 - e^{-0.7 Z_{sun}/y_z}} » 0.54$$

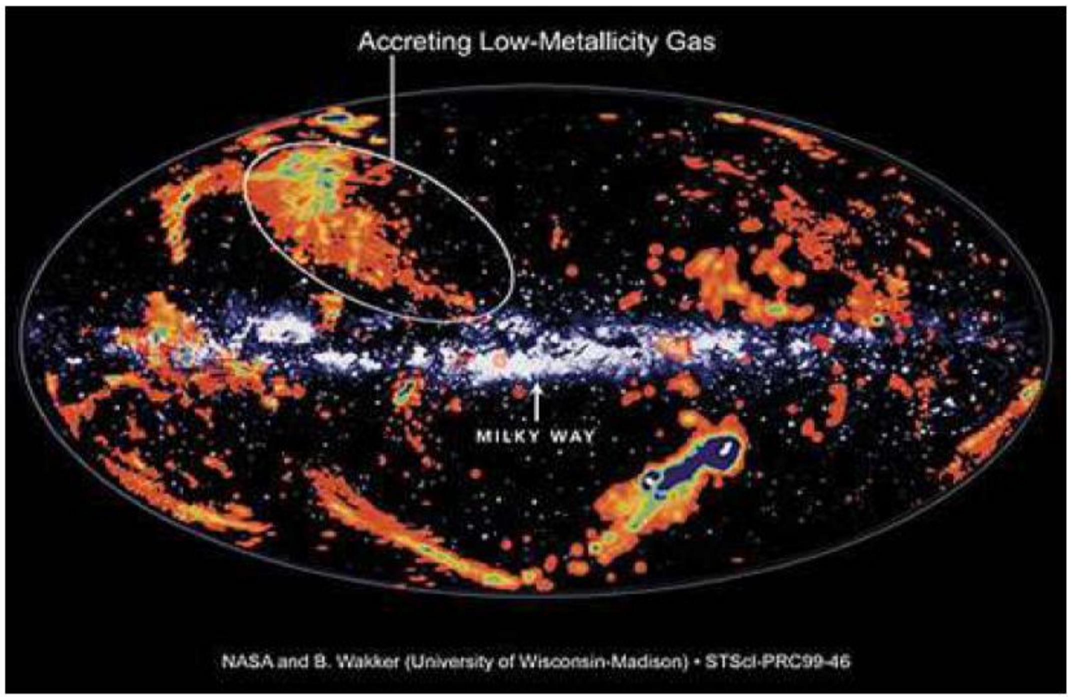
- 50% of the stars in the SN should have metallicities lower than 0.25 times solar. However, only 2% of the G dwarfs show such low metallicities.
- The closed box simple model cannot reproduce either the abundance gradient observed in the Galaxy.

Suggested solutions to the G dwarf problem

1. Stellar migration (Grenon, 1989, 1990)
2. Higher yields in the past (Schmidt 1963)
3. Metal enhanced star formation (Talbott & Arnett 1973, 1975)
4. Prompt initial enrichment (Truran & Cameron 1971)
5. Infall (Oort, 1996; Larson, 1972)
6. Mergers (Nagashima & Okamoto, 2006)

Taking gas flows into account

- The closed system model is not realistic in most cases.
- The are evidences for accretion (`*infall*´) of matter onto the galactic



The extreme infall model

- In this model the gas infall balances the star formation rate ⇒ la the mass in the ISM remains constant.
- In this case: $\dfrac{dM}{dt} = f$

$$\frac{dM_g}{dt} = -Y + E + f = -(1-R)Y + f = 0 \quad \Rightarrow \quad Y = \frac{f}{1-R}$$

$$M_g \frac{dZ}{dt} = Y(1-R)y_z + (Z_f - Z)f = f(y_z + Z_f - Z)$$

$$M_g \frac{dZ}{dt} \bigg/ \frac{dM}{dt} = M_g \frac{dZ}{dM} = \frac{f(y_z + Z_f - Z)}{f} = y_z + Z_f - Z$$

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The solution in the frame of the extreme infall model

- Integrating:

$$\int_{Z_0}^{Z_1} \frac{dZ}{y_z + Z_f - Z} = \int_{M_0}^{M} \frac{dM}{M_g} \quad \Rightarrow \quad ln\left(\frac{y_z + Z_f - Z_0}{y_z + Z_f - Z_1}\right) = \frac{M - M_0}{M_g}$$

$$Z_1 = (y_z + Z_f)(1 - e^{-\nu}) + Z_0 e^{-\nu}$$

- If at galaxy formation $Z_0 = 0$ and $Z_f = 0$ then $Z_1 = y_z(1 - e^{-\nu})$
- And if initially the galaxy was made out of only gas:

$$\nu = \frac{M - M_0}{M_g} = \frac{M - M_0}{M_0} = \mu^{-1} - 1$$

- $\nu \rightarrow \infty$ and $Z_1 \rightarrow y_z$ when $\mu \rightarrow 0$

The Mass-Metallicity relation for galaxies

- We can write the total mass of metal formed during the history of a given galaxy as:

$$ZM = \int_0^t \int_{m_{t'}}^{m_u} m\, p_{z,m}\, \Psi(t' - \tau_m)\, \phi(m, t' - \tau_m)\, dt'\, dm$$

- Using IRA:

$$ZM = \int_0^t \Psi(t)\, dt \int_{m_t}^{m_u} m\, p_{z,m}\, \phi(m)\, dm$$

$$ZM = (1 - R)\, y_z \Psi_T = \frac{M_s}{1 - R}\, y_z (1 - R) \Rightarrow Z \approx \frac{y_z M_s}{M}$$

The mass-metallicity relation for galaxies

- For elliptical galaxies there are two possible alternatives:
  - The SN rate at the beginning was very high thus producing important winds . Since the number of SN should be proportional to M_s, we have:

$$\frac{GMM_g}{R} \approx \frac{GM^2}{R} \approx E_{SN} \propto M_s \Rightarrow \frac{M^2}{R} \propto M_s$$

  - Assuming a relation between mass and radius sucha as: $M \propto R^{\alpha}$

$$\frac{M^2}{R} \propto M^{2-(1/\alpha)} \propto M_s \propto \{M \propto M_s / Z\} \propto M_s$$

  - Hence: $Z \propto M_s^{\frac{\alpha-1}{2\alpha-1}}$
  - To reproduce the observed value of , $Z \propto M_s^{0.25}$, $\alpha = 1.5$

The mass-metallicity relation for galaxies

- Ellipticals form by collisions with other proto-galaxies.Las elípticas se forman mediante colisiones entre protogalaxias.
  - If the star formation efficiency depends of the system mass:

$$\left(\frac{Y}{M_g}\right) \propto M^p \Rightarrow \left(\frac{M_s}{M_g}\right) \propto M^p \Rightarrow M_s \propto M^{p+1}$$

  - Taking into account that $Z \approx \frac{y_z M_s}{M}$

    we get: $M_s \propto M^{p+1} \propto \left(\frac{M_s}{Z}\right)^{p+1} \Rightarrow Z \propto M_s^{\frac{p}{p+1}}$

  - The observed relation $Z \propto M_s^{0.25}$ requieres $p \sim 1/3$
- In both cases $Z_s \rightarrow y_z$ and, at some point, the relation will flatten.

The time scale of chemical evolution

- Time scale for time consumption: in the absence of accretion, How long would it take to consume all the gas?

$$t_* = M_g \Big/ \left|\frac{dM_g}{dt}\right| = \frac{M_g}{1-Y+E} = \frac{M_g}{\left|(1-R)Y\right|}$$

  - In the SN, M_g ~5.7 $M_\odot pc^{-2}$ y la SFR is Ψ=4.2$M_\odot pc^{-2} Gyr^{-1}$. For R ~ 0.4, the gas should be consume in a time scale of τ_* ~ 2 Gyr, much less than a Hubble time.

- Time scale for chemical enrichment:

$$t_Z = Z \Big/ \left|\frac{dZ}{dt}\right| = \frac{Z}{\left|Y(1-R)y_z(Z_f - Z)f\right| / M_g} = \frac{M_g Z}{\left|Y(1-R)y_z\right|} = \frac{t_* Z}{y_z}$$

Other infall models: models with outflow and no infall

- In these models f_{in}= 0 and the outflow is parameterised as:

$$f_{out} = \lambda(1- R)\psi(t)$$

  where $\lambda > 0$.

- Assuming IRA and instantaneous mixing, we have:

$$Z = \frac{y_z}{(1+\lambda)} ln[(1+\lambda)\mu^{-1} - \lambda]$$

- The case λ=0 corresponds to the closed box model

Other infall models: models with infall and no outflow

- In these models f_{out}=0 and the inflow is parameterised as:

$$f_{in} = \Lambda(1-R)\psi(t)$$

where Λ > 0 and Λ ≠ 1

- Assuming IRA and instantaneous mixing, we have:

$$Z = \frac{y_z}{\Lambda}\left[1-\left(\Lambda-(\Lambda-1)\mu^{-1}\right)^{-\Lambda/(1-\Lambda)}\right]$$

- Λ=0 corresponds to the close box model.
- Λ=1 corresponds to the extreme infall model.

Time varying infall

- First introduced by Lynden-Bell.
- His Best Accretion Model (BAM) is "an exactly solvable accretion model in which the gas mass rises to a maximum and then declines" with time.
- In fact, this model gives a rather prompt initial enrichment automatically.

8.5 *Inflow models*

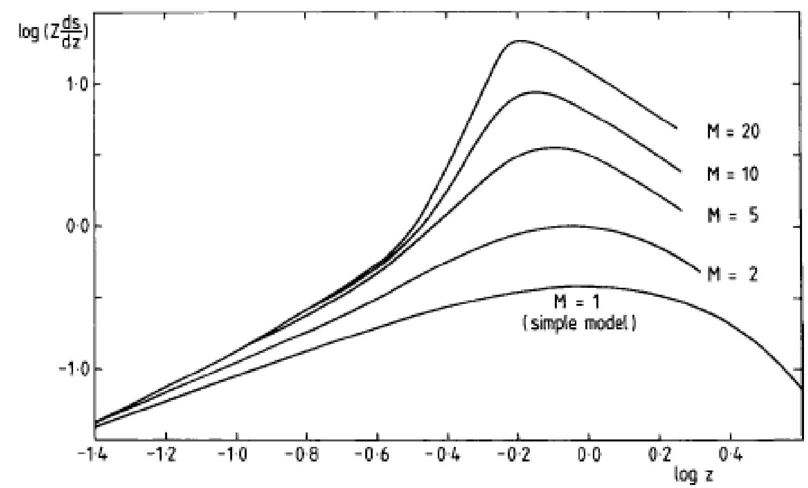


Fig. 8.25. Theoretical abundance distribution functions according to Lynden-Bell's 'Best Accretion Model' for different values of M, after Pagel (1989b).

M (≥ 1) is the final mass of the system in units of its initial mass.

Clayton models

- This is a simple model (it assumes IRA and instantaneous mixing).
- The infall is truncated at some time t_0 and is zero thereafter.
- The SFR is linear with the mass of gas.

$$\frac{dM_g}{dt} = -\psi(1-R) + f$$

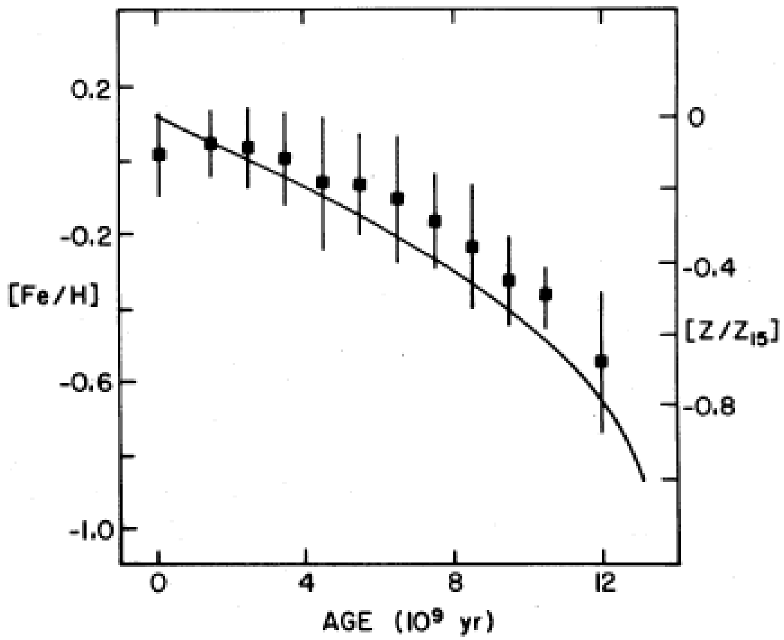
$$\psi(1-R) = \omega M_g \text{, with } \omega = \text{const.}$$

$$M_g = M_{g,0}\, exp\left(-\omega t + \int_o^t \frac{f(t')}{M_g(t')} dt' \right)$$

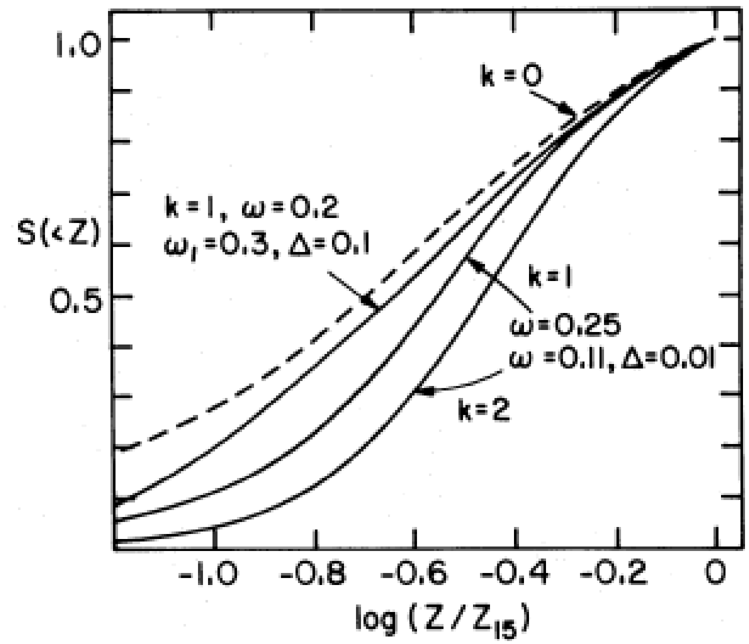
$$\frac{f(t)}{M_g(t)} = \omega_f(t)$$

is the instantaneous rate at which the current $M_g(t)$ is being replenished by the current infall f(t)

Clayton models



Age-metallicity relation



Stellar metallicity distribution

This model does not produce enough metallicity gradient as compared with observations

Model comparison

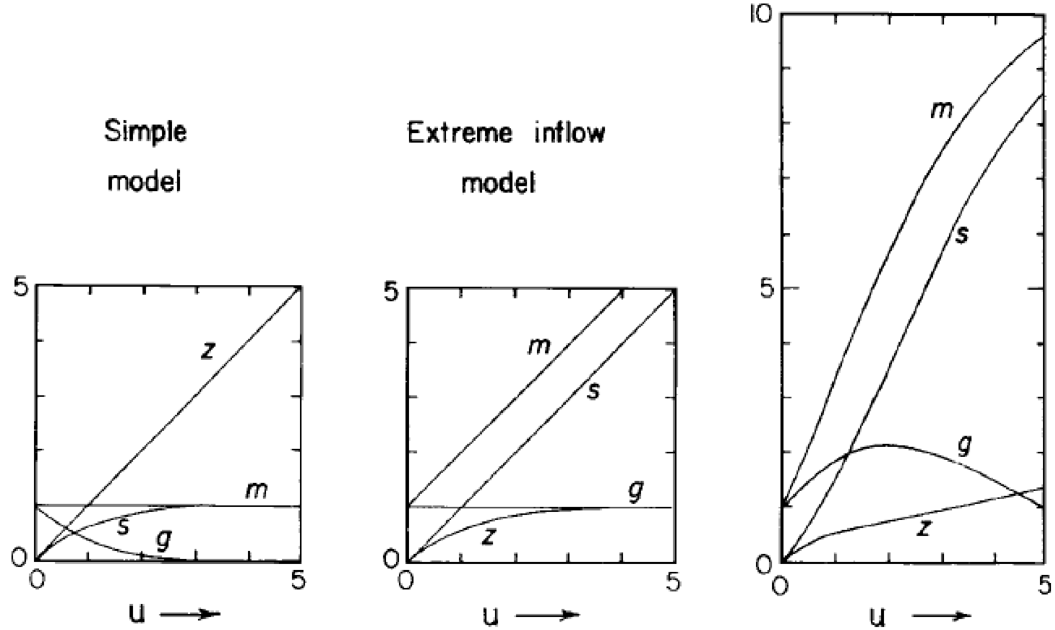


Fig. 8.24. Schematic behaviour of gas mass, total mass and metallicity in the Simple model (left), the extreme inflow model of Larson (1972) (middle) and a model with time-decaying inflow (right). The abscissa is $u \equiv \int_0^t \omega(t')dt'$ where ω is the (constant or otherwise) transition probability per unit time for gas to change into stars. The initial mass has been taken as unity in each case.

Two-zone models : Pagel (1989, Rev. Mex. Astron. Astrof. 18, 161)

AN ANALYTICAL MODEL FOR THE EVOLUTION OF PRIMARY ELEMENTS IN THE GALAXY

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Hartwick's (1976) modified model for the halo and Clayton's (1985, 1988) analytical inflow formalism for the disk are presently used to formulate a self-consistent, analytical chemical-evolution model for the disk and halo of the Galaxy in the solar neighborhood; both formalisms are modified to permit a time delay between the production of such elements as O, and the alpha-elements, r-process elements, and such 'delayed' elements as the s-process products. The model is found to conform well with such traditional constraints as star formation rate behavior and the G dwarf problem; model predictions also suggest that enrichment, as a function of time, is an important effect, and that the s-process is 'primary'.

A two-zone model for The Galaxy

- Is a model with two zones: disc + halo.
- The gas falls from the halo onto the disc at a rate: $\omega(t) = cte = f(R)$
- Defining the dimensionless variable $u = \int_0^t \omega(t')dt'$

we have the equations:

$$\frac{ds}{dt} = \omega g \Rightarrow \frac{ds}{du} = g \Rightarrow$$

$$\frac{dg}{du} + g = \frac{F}{\omega}$$

$$g(u) = e^{-u}\left(g_0 + \int_0^u \frac{F}{\omega} e^{u'} du'\right)$$

where s stands for mass in stars, g stands for mass in gas, F is the mass inflow and g_0 is the initial mass of the systems before any stars were formed.

Pagel and Tautvaisiene, 1995
MNRAS, 276, 505

- Each stellar generation ejects a mass fraction, q_1, of newly synthesised elements and a mass fraction, q_2, after a time delay Δ. $p_1 = q_1/\alpha$ and $p_2 = q_2/\alpha$ are their respective yields and

$$\frac{d}{du}(gZ) + gZ = \begin{cases} p_1 g(u) & u < w\Delta \\ p_1 g(u) + p_2 g(u - w\Delta) & u \geq w\Delta \end{cases}$$

  - *Note: α is the locked mass fraction (1-R) in the IRA.*

Pagel and Tautvaisiene

- Taking Z = $Z_1 + Z_2$ where Z_1 and Z_2 represent instantaneous and delayed production we have:

$$\frac{dZ_1}{du} + \frac{F}{wg} Z_1 = p_1$$

$$\frac{dZ_2}{du} + \frac{F}{wg} Z_2 = \begin{cases} 0 & u < wD \\ p_2 \dfrac{g(u - wD)}{g(u)} & u \geq wD \end{cases}$$

- To reproduce the oxygen abundance distribution of G dwarfs, a functional form of infall of the type: $\frac{F}{\omega g} = \frac{K}{u + u_0}$ is assumed, with K = 3, u_0 = 1.3
- La composición del gas que cae es primordial hasta un tiempo u_1= 0.14, a partir del cual p_1^O = 0.7 Z_{sol} .
- La evolución del disco comienza cuando f_0^O = 0.08.

Pagel & Tautvaisiene's goals

Our model will aim to satisfy the following constraints:-

1. To provide a consistent story of the development of the halo and the disk, taking into account initial enrichment of the disk by prior activity in the halo (cf. Kumai et al. 1988).

2. Relative numbers of disk and halo stars in the solar cylinder.

3. The present-day gas fraction in the disk (Kulkarni and Heiles 1987).

4. The ratio of present star formation rate to average past SFR (Scalo 1986).

5. The age-metallicity relation in the disk.

6. The metallicity distribution functions in the halo and disk (G-dwarf problem).

7. Variations in relative abundances of primary elements (assuming s-process, r-process both primary).

8. To explain the presence of abundance gradients.

Pagel & Tautvaisiene's results

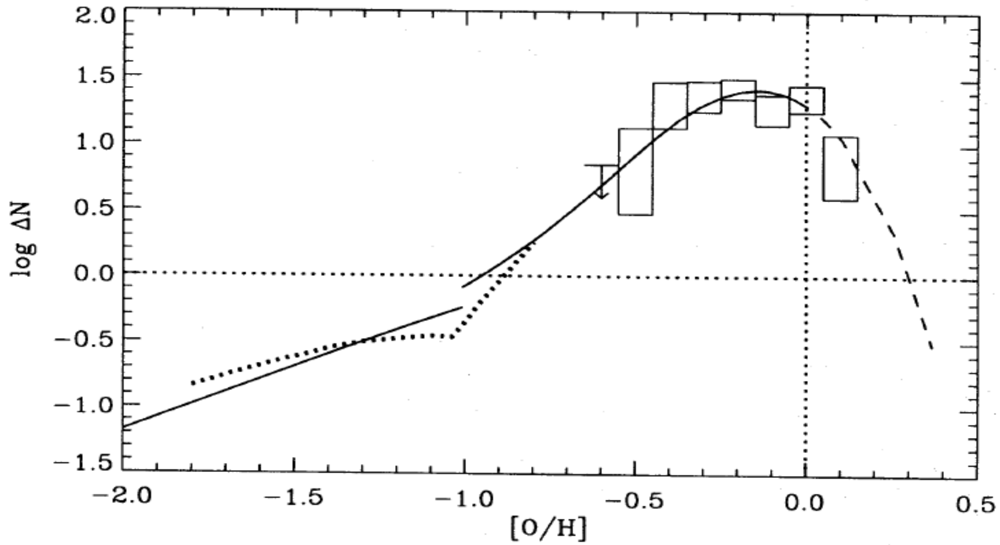


Figure 1. Oxygen ADF for G dwarfs in the solar cylinder, normalized to 132 stars after Pagel & Patchett (1975) and binned in intervals of 0.1 in [O/H] (cf. Pagel 1989b). Boxes show observed numbers with error limits after Sommer-Larsen (1991) and the dotted curves on the left side of the diagram show the extension to the metal-weak thick disc after Beers & Sommer-Larsen (1995). The solid curve shows the ADF from our model assuming a present-day gas fraction $\mu \equiv g/m = 0.11$; its extension to [O/H] > 0, shown by the broken curve on the right, represents where it would go if lower values of μ were allowed.

Pagel & Tautvaisiene's results

Table 1. Basic model parameters.

Parameter	Value
k	3
u_0	1.3
u_1	0.14
$m(\infty)$	7.07
$u_\odot$	3.1
u_{now}	4.5
Age (Gyr)	15
$p^O/Z^O_\odot$	0.70

$u_\odot$ is the value of u at formation of the Solar system.
u_{now} is the local value of u at the present day.
For the inner disc ($R_m < 7$ kpc) we assume age 16.5 Gyr, $u'_{now} = 7.43$

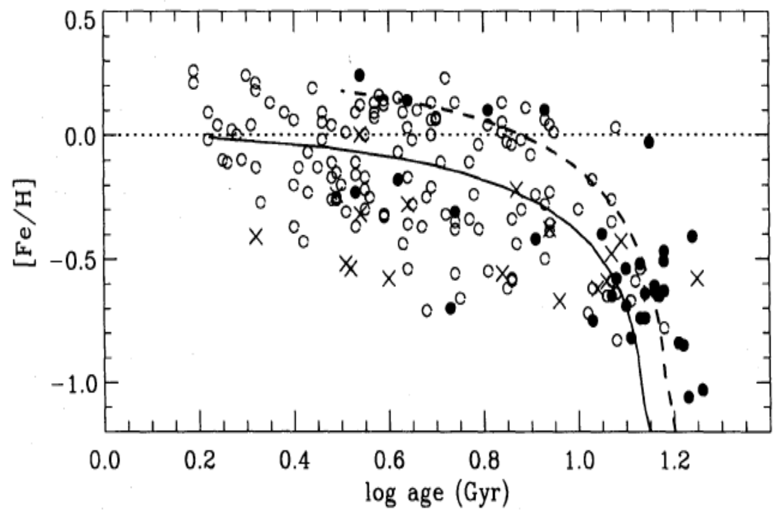


Figure 2. Age-metallicity relation from our model, assuming ω = constant = 0.3 Gyr^{-1} with an age of 15 Gyr for $R_m > 7$ kpc (full-drawn curve) and $\omega = 0.45$ with an age of 16.5 Gyr for the inner Galactic disc ($R_m < 7$ kpc: broken curve) compared with data from Edvardsson et al. (1993). Open circles: stars with mean galactocentric distance R_m between 7 and 9 kpc (similar to the Sun); filled circles: stars from the inner disc ($R_m < 7$ kpc); crosses: stars from the outer disc ($R_m \geq 9$ kpc). Typical error bars are ±0.1 dex in each coordinate.

Pagel & Tautvaisiene's results

Evolution of elements in the Galactic disc 509

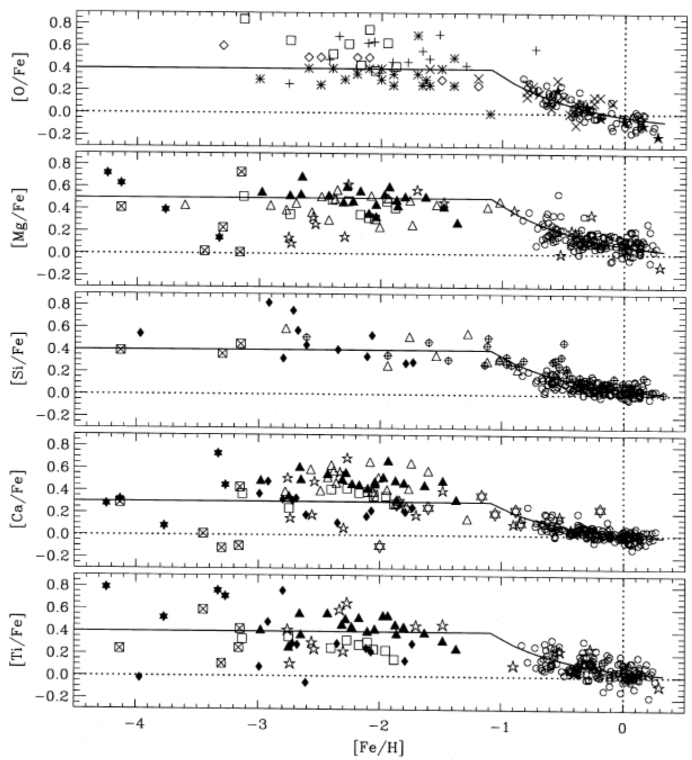


Figure 3. Oxygen, magnesium, silicon, calcium and titanium: iron abundance ratios plotted against metallicity [Fe/H] for nearby disc stars and extension to [Fe/H] = −4.5 (halo and metal-weak thick disc). Symbols indicate various data sources. Open squares: Nissen et al. (1994); squares with crosses: Primas, Molaro & Castelli (1994); open circles: Edvardsson et al. (1993) for $R_m \geq 7$ kpc; 'plus' signs: King (1993); filled six-cornered 'stars': Norris et al. (1993); open rhombs: Bessell, Sutherland & Ruan (1991); field triangles: Magain (1989); open triangles: Magain (1987); open five-cornered 'stars': Tautvaišienė & Straižys (1989); crosses: Barbuy & Erdelyi-Mendez (1989); asterisks: Barbuy (1988); filled rhombs: Gratton & Sneden (1988); open six-cornered 'stars': Hartmann & Gehren (1988); circles with 'plus' signs: François (1986); filled five-cornered 'stars': Kyröläinen et al. (1986).

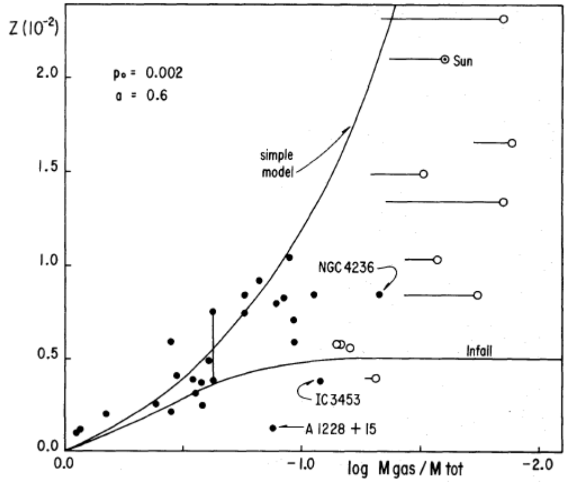
Models with metallicity dependent yields

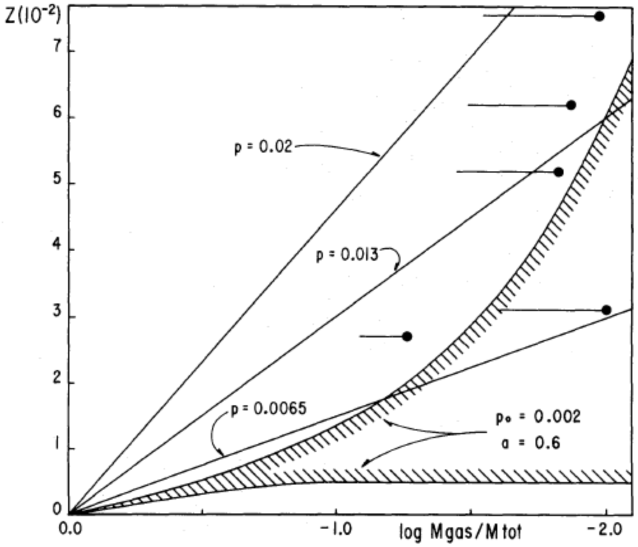
Peimbert & Serrano 1982, MNRAS,276, 505

In this model it is assumed that $y = y(Z)$

$$y = y_0 + aZ$$

$$Z = \frac{p_0}{1-a}\left\{1 - exp\left[(1 - \frac{Mtot}{Mg})(1-a)\right]\right\}$$





Abundance gradient production

- Any model must reproduce the observed gradients.
- From a theoretical point of vie, we can analyse directly the differential equations:

$$\frac{\partial \mu}{\partial t} = f(r,t) - \frac{1}{r}\frac{\partial}{\partial r}(rvg)$$

$$\frac{\partial s}{\partial t} = \Psi(r,t) - E(r,t)$$

$$\frac{\partial g}{\partial t} = -\Psi(r,t) + E(r,t) + f(r,t) - \frac{1}{r}\frac{\partial}{\partial r}(rvg)$$

- con

$$f(r,t) = f_0 \frac{\exp(-t/\tau_f)}{\tau_f (1-\exp(-T_G/\tau_f))}$$

$$f_0 = f_0(r_0)\exp\frac{(r_0 - r)}{r_0}$$

$$\Psi(r,t) = c_n g^n(r,t)$$

and obtain the time derivative of the gradient:

$$\boxed{\begin{aligned}\frac{\partial \ln Z}{\partial t} = \frac{\alpha y \Psi}{gZ}\left(\frac{-\partial \ln Z}{\partial r} + \frac{d\ln(\alpha y)}{dr} + \frac{\partial \ln(\Psi/g)}{\partial r}\right) \\ -\frac{f}{g}\frac{\partial \ln(f/g)}{\partial r} + \frac{\partial}{\partial r}\left(\frac{Z^f f}{Zg}\right) \\ -\frac{dv}{dr}\frac{\partial \ln Z}{\partial r} - v\frac{\partial^2 \ln Z}{\partial r^2}\end{aligned}}$$

Variación radial en SFR

Variación radial en FIM o yields

Variación radial en Z infall

Variación radial en infall

Efectos de flujos radiales de entrada o salida

How to produce abundance gradients

The problem in understanding abundance gradients is that there are far too many possible hypotheses to account for them. These include the following:-

1. Variation in the true yield due to the IMF (Güsten and Mezger 1982). If the IMF is bimodal and is cut off or scaled down below 1 solar mass or so, then all the yields can be enhanced by the same factor. With a higher cutoff, they can be enhanced by different factors and you can get more or less what you like according to what you assume. This does not mean that such effects are not significant in reality, however.

2. Variation in effective yield due to continuous or sporadic ejection of hot gas (Pantelaki and Clayton 1987). The alternative idea of a terminal galactic wind, in which the interstellar medium is mixed and gradually heated up until it all escapes (Tinsley and Larson 1979; Arimoto and Yoshii 1987), hardly seems applicable to the solar neighbourhood.

3. Variation in gas fraction, other things being equal. This is the original idea of the "Simple" closed model (Searle and Sargent 1972) and we can examine its consequences by changing ω, leaving k, ωt_0 (which fix the final mass multiplication factor) and Δ unaltered, and evolving the model for a fixed time of 15 Gyrs.

4. Variation in ratio of star formation rate to inflow rate (Diaz and Tosi 1984). We can investigate this again, in our model, by reducing k and/or increasing t_o, and evolving to the same gas fraction after 15 Gyr.

5. Inward gas flows caused by inflow of material with low angular momentum (Mayor and Vigroux 1981; Lacey and Fall 1985; Pitts and Tayler 1989).

6. Gas flows caused by viscous transfer of angular momentum across the disk (Clarke 1989; Sommer-Larsen and Yoshii 1989). These can be both outward and inward at different times and the consequences depend on the viscosity law which is assumed ad hoc and justified by its success in explaining the surface density and angular momentum distributions of galactic disks. In Clarke's model the variation in gas fraction also plays a significant role.

Edmunds' Theorems

T(1) In a model with outflow, but no inflow, the effective yield is always less than that of the simple model.

T(2) In a model with inflow whose metallicity does not exceed that of the system itself, the effective yield is always less than that of the simple model provided the outflow rate exceeds the inflow rate.

T(3) In a model with inflow of unenriched gas, the effective yield is always less than that of the simple model.